



25. Let  $n$  be the smallest integer for which  $7n$  has 2016 digits.  
What is the units digit of  $n$ ?

- A 0                      B 1                      C 4                      D 6                      E 8

1695



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25. **D** For  $n$  to be the smallest integer for which  $7n$  has 2016 digits,  $7n$  must start with 1, be followed by 2014 zeros and end with a digit  $a$ . When this number is divided by 7, the answer is formed from the repeating sequence of 6 digits 142857. The remainders also form a repeating sequence 3, 2, 6, 4, 5, 1. These sequences are repeated 335 times as  $6 \times 335$  is 2010. The last 4 zeros (to make 2014 zeros in total) and the final  $a$  create the last section of the division as shown:

$$\begin{array}{r} \dots\dots\dots 1\ 4\ 2\ 8 \\ \hline \dots\ 10^3 0^2 0^6 0^4 a \end{array}$$

Finally,  $40 + a$  must be divisible by 7 and be as small as possible. So  $a = 2$  and as  $42 \div 7 = 6$  the units digit of  $n$  is 6.