



19. Let  $S$  be a set of five different positive integers, the largest of which is  $m$ . It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of  $S$ . What is the smallest possible value of  $m$ ?
- A 2                      B 4                      C 9                      D 11                      E 12

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19. **D** Let  $S$  consist of  $h, j, k, l, m$  in ascending order of size. We want  $m$  to be as small as possible. Given three side-lengths, there is a quadrilateral with non-zero area with a specified fourth side-length if and only if the fourth side-length is less than the sum of the other three side-lengths. To ensure that  $j, k, l, m$  are not the side-lengths of such a quadrilateral, we must have  $m \geq j + k + l$ . Likewise, considering  $h, j, k, l$ , we must have  $l \geq h + j + k$ . Since the smallest possible values of  $h, j$  and  $k$  are 1, 2 and 3 respectively then  $l \geq 1 + 2 + 3$  so 6 is the smallest value of  $l$ . Also  $m \geq 2 + 3 + 6$  so 11 is the smallest value of  $m$ .