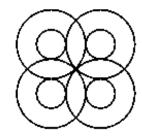




The diagram shows eight circles of two different sizes. The circles are 17. arranged in concentric pairs so that the centres form a square. Each larger circle touches one other larger circle and two smaller circles. The larger circles have radius 1. What is the radius of each smaller circle?



A  $\frac{1}{3}$  B  $\frac{2}{5}$  C  $\sqrt{2}-1$  D  $\frac{1}{2}$  E  $\frac{1}{2}\sqrt{2}$ 





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17. Let the radius of each of the smaller circles be r and let the centres of the circles be A, B, C C and D in order. We are given that ABCD is a square. When two circles touch externally, the distance between their centres equals the sum of their radii. Hence AB and BC have length r+1 and AC has length 1+1=2. By Pythagoras' Theorem  $(r+1)^2+(r+1)^2=2^2$ , so  $2(r+1)^2=2^2=4$  and therefore  $(r+1)^2=2$ . Square rooting both sides gives  $r+1=\sqrt{2}$ , as we must take the positive root, and so  $r=\sqrt{2}-1$ .