



10. The positive integer n is between 1 and 20. Milly adds up all the integers from 1 to n inclusive. Billy adds up all the integers from n + 1 to 20 inclusive. Their totals are the same. What is the value of n?

A 11

B 12

C 13

D 14

E 15

1580



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10. **D** The total of the numbers from 1 to 20 is  $\frac{1}{2} \times 20 \times (20 + 1) = 210$ . If Milly and Billy have totals which are equal, their totals must each be 105. Milly's total, of the numbers from 1 to n, is  $\frac{1}{2}n(n+1)$  so  $\frac{1}{2}n(n+1) = 105$  which gives  $n^2 + n = 210$ . Therefore  $n^2 + n - 210 = 0$  which factorises to give (n + 15)(n - 14) = 0. As n is a positive integer, n = 14.