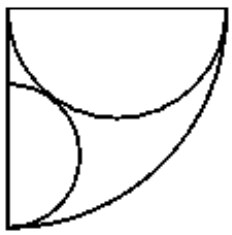
19. The diagram shows a quadrant of radius 2, and two touching semicircles. The larger semicircle has radius 1. What is the radius of the smaller semicircle?



A $\frac{\pi}{6}$ B $\frac{\sqrt{3}}{2}$ C $\frac{1}{2}$ D $\frac{1}{\sqrt{3}}$ E $\frac{2}{3}$

---

19. E Let the centre of the quadrant be O, the centre of the larger semicircle be A and the centre of the smaller semicircle be B. Let the radius of the smaller semicircle be r. It is given that $OA = 1$. The common tangent to the two semicircles at the point of contact makes an angle of $90°$ with the radius of each semicircle. Therefore the line AB passes through the point of contact, as $2 \times 90° = 180°$ and angles on a straight line sum to $180°$. So the line AB has length $r + 1$. This is the hypotenuse of the right-angled triangle OAB in which $OA = 1$ and $OB = 2 - r$. By Pythagoras' Theorem $(2 - r)^2 + 1^2 = (r + 1)^2$, so $4 - 4r + r^2 + 1 = r^2 + 2r + 1$ and therefore $4 = 6r$ and so $r = \frac{2}{3}$.