



22. Consider numbers of the form 10n + 1, where n is a positive integer. We shall call such a number 'grime' if it cannot be expressed as the product of two smaller numbers, possibly equal, both of which are of the form 10k + 1, where k is a positive integer. How many 'grime numbers' are there in the sequence 11, 21, 31, 41, ..., 981, 991?

A 0

B 8

C 87

D 92

E 99

1392



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C The numbers in the sequence 11, 21, 31, 41, ..., 981, 991 are of the form 10n + 1 for n = 1 to 99. There are therefore 99 numbers in this sequence.
Twelve terms of this sequence can be expressed using factors of the form 10k + 1. In this form, these terms are 11 × 11, 11 × 21, 11 × 31, ..., 11 × 81 and 21 × 21,

 $21 \times 31$ ,  $21 \times 41$  and  $31 \times 31$ . All other pairings give products that are too large. Hence, there are 99 - 12 = 87 'grime' numbers.