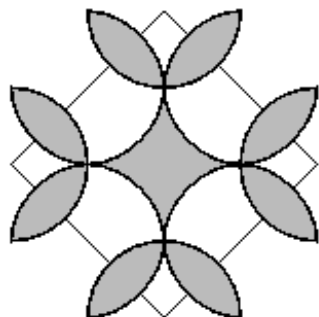




21.



The shaded design shown in the diagram is made by drawing eight circular arcs, all with the same radius. The centres of four arcs are the vertices of the square; the centres of the four touching arcs are the midpoints of the sides of the square. The diagonals of the square have length 1.

What is the total length of the border of the shaded design?

- A 2π B $\frac{5\pi}{2}$ C 3π D $\frac{7\pi}{2}$ E 4π

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- 21. B** Let the top vertex of the square be A and the midpoints of the two lines that meet at A be B and C . The line BC is of length $\frac{1}{2}$ and is perpendicular to the diagonal of the square through A . Let the point of intersection of these two lines be D . Let the end of the uppermost arc, above B , be E . Then $ADBE$ is a rhombus, made from four radii of the arcs, AD , DB , BE and EA , each of length $\frac{1}{4}$. As $\angle ADB = 90^\circ$, this rhombus is a square. It then follows that the four arcs whose centres are the vertices of the original square are all semi-circles. The remaining four touching arcs are each $\frac{3}{4}$ of a circle. In total, the length of the border is $4 \times \frac{1}{2} + 4 \times \frac{3}{4}$ times the circumference of a circle with the same radius, so is $5 \times 2\pi \times \frac{1}{4} = \frac{5}{2}\pi$.