



20. The ratio of two positive numbers equals the ratio of their sum to their difference. What is this ratio?
- A $(1+\sqrt{3}):2$ B $\sqrt{2}:1$ C $(1+\sqrt{5}):2$ D $(2+\sqrt{2}):1$ E $(1+\sqrt{2}):1$

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20. **E** Let the two positive numbers be x and y with $x > y$. The sum of the numbers is greater than their difference, so the two ratios which are equal are $x : y$ and $x + y : x - y$. Therefore $\frac{x}{y} = \frac{x+y}{x-y}$. By dividing the top and bottom of the right-hand side by y we obtain $\frac{x}{y} = \frac{\frac{x}{y} + 1}{\frac{x}{y} - 1}$. Letting $k = \frac{x}{y}$ gives $k = \frac{k+1}{k-1}$ which gives the quadratic $k^2 - 2k - 1 = 0$. Completing the square gives $(k-1)^2 = 2$ whence $k = 1 \pm \sqrt{2}$. However, as x and y are both positive, $k \neq 1 - \sqrt{2}$. As the ratio $\frac{x}{y} = 1 + \sqrt{2}$, the ratio $x : y$ is $1 + \sqrt{2} : 1$.