25. How many distinct pairs (x, y) of real numbers satisfy the equation $(x+y)^2 = (x+4)(y-4)$?

A 0 B 1 C 2 D 3 E 4

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25. **B** Starting with $(x + y)^2 = (x + 4)(y - 4)$ and expanding both sides gives $x^2 + 2xy + y^2 = xy - 4x + 4y - 16$, i.e. $x^2 + (y + 4)x + y^2 - 4y + 16 = 0$. To eliminate the xy term we let $z = x + \frac{1}{2}y$ and then replace x by $z - \frac{1}{2}y$. The equation above becomes $z^2 + 4\left(z - \frac{1}{2}y\right) + \frac{3}{4}y^2 - 4y + 16 = 0$. However,

$$z^2 + 4\left(z - \tfrac{1}{2}y\right) + \tfrac{3}{4}y^2 - 4y + 16 = (z + 2)^2 + \tfrac{3}{4}y^2 - 6y + 12$$

$$= (z + 2)^2 + \tfrac{3}{4}\left(y^2 - 8y + 16\right) = (z + 2)^2 + \tfrac{3}{4}(y + 4)^2.$$

So the only real solution is when $z = -2$ and $y = 4$; i.e. $x = -4$ and $y = 4$.