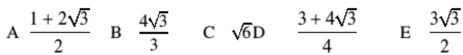
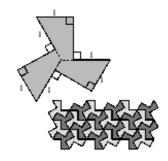




24. The top diagram on the right shows a shape that tiles the plane, as shown in the lower diagram. The tile has nine sides, six of which have length 1. It may be divided into three congruent quadrilaterals as shown. What is the area of the tile?



$$\frac{3+4\sqrt{3}}{4}$$



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В The diagram shows one of the three quadrilaterals making up the tile, labelled and with a line BE inserted. Note that it is a trapezium. As three quadrilaterals fit together, it may be deduced that  $\angle ABC = 360^{\circ} \div 3 = 120^{\circ}$ , so  $\angle BAD = 60^{\circ}$ . It may also be deduced that the length of AB is 1 + x, where x is the length of BC. Now  $\cos \angle BAD = \cos 60^\circ = \frac{1}{2} = \frac{1-x}{1+x}$ . So 1 + x = 2 - 2x, i.e.  $x = \frac{1}{3}$ . The area of ABCD is  $\frac{1}{2}(AD + BC) \times CD = \frac{1}{2}(1 + \frac{1}{3}) \times \frac{4}{3}\sin 60^{\circ}$  $=\frac{2}{3}\times\frac{4}{3}\times\frac{\sqrt{3}}{2}=\frac{4\sqrt{3}}{9}$ . So the area of the tile is  $3\times\frac{4\sqrt{3}}{9}=\frac{4\sqrt{3}}{3}$ .

