23. Tom and Geri have a competition. Initially, each player has one attempt at hitting a target. If one player hits the target and the other does not then the successful player wins. If both players hit the target, or if both players miss the target, then each has another attempt, with the same rules applying. If the probability of Tom hitting the target is always $\frac{4}{5}$ and the probability of Geri hitting the target is always $\frac{2}{3}$, what is the probability that Tom wins the competition?

A $\frac{4}{15}$ B $\frac{8}{15}$ C $\frac{2}{3}$ D $\frac{4}{5}$ E $\frac{13}{15}$

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23. **C** Tom wins after one attempt each if he hits the target and Geri misses. The probability of this happening is $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$. Similarly the probability that Geri wins after one attempt is $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$. So the probability that both competitors will have at least one more attempt is $1 - \frac{4}{15} - \frac{2}{15} = \frac{3}{5}$.

Therefore the probability that Tom wins after two attempts each is $\frac{3}{5} \times \frac{4}{15}$. The probability that neither Tom nor Geri wins after two attempts each is $\frac{3}{5} \times \frac{3}{5}$. So the probability that Tom wins after three attempts each is $\left(\frac{3}{5}\right)^2 \times \frac{4}{15}$ and, more generally, the probability that he wins after n attempts each is $\left(\frac{3}{5}\right)^{n-1} \times \frac{4}{15}$.

Therefore the probability that Tom wins is $\frac{4}{15} + \left(\frac{3}{5}\right) \times \frac{4}{15} + \left(\frac{3}{5}\right)^2 \times \frac{4}{15} + \left(\frac{3}{5}\right)^3 \times \frac{4}{15} + \ldots$.

This is the sum to infinity of a geometric series with first term $\frac{4}{15}$ and common ratio $\frac{3}{5}$. Its value is $\frac{4}{15} \div \left(1 - \frac{3}{5}\right) = \frac{2}{3}$.