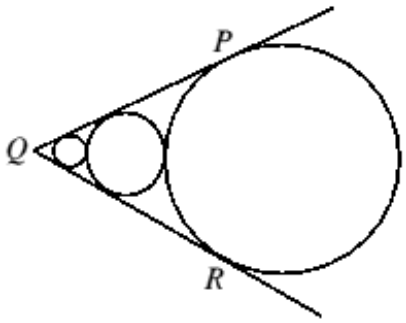
24. Three circles and the lines PQ and QR touch as shown. The distance between the centres of the smallest and the biggest circles is 16 times the radius of the smallest circle. What is the size of $\angle PQR$?

A 45° B 60° C 75° D 90° E 135°



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24. B Let the radii of the circles from smallest to largest be r_1, r_2 and r_3 respectively. Hence $16r_1 = r_3 + 2r_2 + r_1$, thus $r_3 = 15r_1 - 2r_2 \ldots$ (1). Let $r_1 + x$ be the distance from Q to the centre of the smallest circle. By similar triangles,

$$\frac{r_1}{r_1 + x} = \frac{r_2}{x + 2r_1 + r_2} = \frac{r_3}{16r_1 + r_1 + x} \ldots (2).$$

Thus $r_1(x + 2r_1 + r_2) = r_2(r_1 + x)$. Hence $r_2 = \dfrac{r_1x + 2r_1^2}{x} \ldots$ (3). From (1) and (2) $\dfrac{r_1x}{r_1 + x} = \dfrac{(15r_1 - 2r_2)x}{17r_1 + x}$ hence $\dfrac{r_1x}{r_1 + x} = \dfrac{15r_1x - 2(r_1x + 2r_1^2)}{17r_1 + x}$. Dividing throughout by r_1 and simplifying gives $12x^2 - 8r_1x - 4r_1^2 = 0$. Hence $(3x + r_1)(x - r_1) = 0$ so, as $r_1 > 0$, $x = r_1$. Thus $\sin\dfrac{\angle PQR}{2} = \dfrac{r_1}{r_1 + x} = \dfrac{r_1}{2r_1} = \dfrac{1}{2}$. Hence $\frac{1}{2}\angle PQR = 30°$ so $\angle PQR = 60°$.