



22. If  $x^2 - px - q = 0$ , where p and q are positive integers, which of the following could not equal  $x^3$ ?

A 4x + 3

B 8x + 5 C 8x + 7

D 10x + 3

E 26x + 5

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Since  $x^2 - px - q = 0$ , it follows that  $x^3 = px^2 + qx$ . 22. В

But  $x^2 = px + q$  and so  $x^3 = p(px + q) + qx$ , ie  $x^3 = (p^2 + q)x + pq$ .

The three possible values shown for pq are 3, 5 and 7.

If pq = 3,  $p^2 + q = 1^2 + 3 = 4$  or  $p^2 + q = 3^2 + 1 = 10$ . Hence 4x + 3 and 10x + 3 could equal  $x^3$ .

If pq = 7, we may take p = 1, q = 7 to get  $p^2 + q = 1^2 + 7 = 8$ . Hence 8x + 7could equal  $x^3$ .

If pq = 5, we may take p = 5, q = 1 to get  $p^2 + q = 5^2 + 1 = 26$ . Hence 26x + 5could equal  $x^3$ .

However, the only other possibility, p = 1, q = 5 gives  $p^2 + q = 6 \neq 8$ . Therefore  $8x + 5 \neq x^3$ .