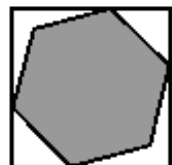




21. The diagram shows a regular hexagon, with sides of length 1, inside a square. Two vertices of the hexagon lie on a diagonal of the square and the other four lie on the edges.



What is the area of the square?

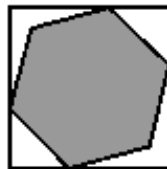
- A  $2 + \sqrt{3}$       B 4      C  $3 + \sqrt{2}$       D  $1 + \frac{3\sqrt{3}}{2}$       E  $\frac{7}{2}$

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21. A The hypotenuse of one of the small right-angled triangles is parallel to the diagonal and hence makes angles of  $45^\circ$ . Since the hypotenuse has length 1, the other two sides have length  $\frac{1}{\sqrt{2}}$ , by Pythagoras' Theorem. As the internal angle of a regular hexagon is  $120^\circ$ , drawing a diagonal from NW to SE forms two triangles, bottom right, each with angles  $45^\circ$ ,  $120^\circ$  and  $15^\circ$ . (The sum of the angles in a triangle is  $180^\circ$ ).



Let the square have length  $y$  units. Using the Sine Rule gives  $\frac{y - \frac{1}{\sqrt{2}}}{\sin 120^\circ} = \frac{1}{\sin 45^\circ}$ .

Hence  $y - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2} \div \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$  and therefore  $y = \frac{\sqrt{3} + 1}{\sqrt{2}}$ .

Hence the area of the square is  $y^2 = \left(\frac{\sqrt{3} + 1}{\sqrt{2}}\right)^2 = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$ .