



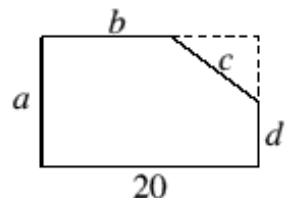
20. A triangle is cut from the corner of a rectangle. The resulting pentagon has sides of length 8, 10, 13, 15 and 20 units, though not necessarily in that order. What is the area of the pentagon?
- A 252.5      B 260      C 270      D 275.5      E 282.5

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20. C The diagram shows the original rectangle with the corner cut from it to form a pentagon. It may be deduced that the length of the original rectangle is 20 and that  $a, b, c, d$  are 8, 10, 13, 15 in some order.



By Pythagoras' Theorem  $c^2 = (20 - b)^2 + (a - d)^2$ . So  $c$  cannot be 8 as there is no right-angled triangle having integer sides and hypotenuse 8. If  $c = 10$ , then  $(20 - b)$  and  $(a - d)$  are 6 and 8 in some order, but this is not possible using values of 8, 13 and 15. Similarly, if  $c = 15$ , then  $(20 - b)$  and  $(a - d)$  are 9 and 12 in some order, but this is not possible using values of 8, 10 and 13. However, if  $c = 13$ , then  $(20 - b)$  and  $(a - d)$  are 5 and 12 in some order, which is true if and only if  $a = 15, b = 8, d = 10$ .

So the area of the pentagon is  $20 \times 15 - \frac{1}{2} \times 5 \times 12 = 270$ .