25. X is a positive integer in which each digit is 1; that is, X is of the form 11111… .
Given that every digit of the integer $pX^2 + qX + r$ (where p, q and r are fixed integer coefficients and $p > 0$) is also 1, irrespective of the number of digits X, which of the following is a possible value of q?

A -2 B -1 C 0 D 1 E 2

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25. E Let X consist of x digits, each of which is 1. So $X = \frac{10^x - 1}{9}$. Let $pX^2 + qX + r$ consist of y digits, each of which is 1. So $pX^2 + qX + r = \frac{10^y - 1}{9}$. Then $p\left(\frac{10^x-1}{9}\right)^2 + q\left(\frac{10^x-1}{9}\right) + r = \frac{10^y-1}{9}$, that is $p(10^{2x} - 2 \times 10^x + 1) + 9q(10^x - 1) + 81r = 9(10^y - 1)$, that is (on dividing throughout by 10^{2x}) $p + (9q - 2p)10^{-x} + (p - 9q + 81r)10^{-2x} = 9 \times 10^{y-2x} - 9 \times 10^{-2x}$.
We now let x tend to infinity (through integer values). The LHS of the above equation tends to p, and the second term on the right goes to 0. By continuity of the function $f(u) = 10^u = e^{u \log 10}$, we can deduce that $y - 2x$ must tend to a limit. Let this limit be L. Since $y - 2x$ is always an integer, it must actually equal L for all x sufficiently large. Passing to the limit, therefore, we obtain $p = 9 \times 10^L$. Since p is to be an integer, we must have that L (also an integer) is a non-negative integer. Substituting for p in the previous equation and simplifying leads to

$$9q - 18 \times 10^L + (9 \times 10^L - 9q + 81r)10^{-x} = -9 \times 10^{-x}.$$

Passing to the limit again leads to $q = 2 \times 10^L$ and the previous line then also gives $9 \times 10^L - 18 \times 10^L + 81r = -9$. So $r = \frac{10^L - 1}{9}$.
Possible values of (p, q, r) therefore are (9, 2, 0), (90, 20, 1), (900, 200, 11), etc. So of the values given in the question for q, only $q = 2$ is possible.
(*Observe that the three triples above correspond to* $L = 0, L = 1, L = 2$ *respectively and we note that increasing* L *by* 1 *corresponds to multiplying* $pX^2 + qX + r$ *by* 10 *and adding* 1. *As* $pX^2 + qX + r$ *consists only of* 1s, $10(pX^2 + qX + r) + 1$ *will also consist only of* 1s, *explaining why there is an infinite family of quadratics which satisfy the required condition.*)