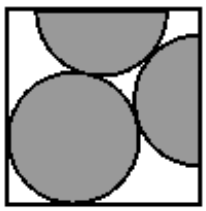
23. In the diagram, the circle and the two semicircles have radius 1. What is the perimeter of the square?



A $6 + 4\sqrt{2}$ B $2 + 4\sqrt{2} + 2\sqrt{3}$ C $3\sqrt{2} + 4\sqrt{3}$
D $4 + 2\sqrt{2} + 2\sqrt{6}$ E 12

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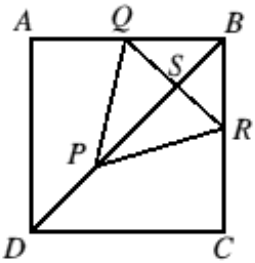
23. D Let the vertices of the square be A, B, C, D and the centres of the circle and the two semicircles be P, Q, R, as shown. The midpoint of QR is S. By symmetry, P and S both lie on diagonal BD of square $ABCD$ and the whole figure is symmetrical about BD.



As P is distance 1 from both AD and DC, the length of DP is $\sqrt{2}$.
As the circles and semicircles are mutually tangent, PQR is an equilateral triangle of side 2, so the length of PS is $\sqrt{3}$. As angles QBS and BSQ are 45° and 90° respectively, triangle SBQ is isosceles, so $SB = SQ = 1$. Hence the length of BD is $\sqrt{2} + \sqrt{3} + 1$. Now the length of the side of the square is $BD \div \sqrt{2}$ so the perimeter of the square is $4 \times (BD \div \sqrt{2})$, that is $2\sqrt{2} \times BD$.
So the perimeter is $2\sqrt{2}(\sqrt{2} + \sqrt{3} + 1)$, that is $4 + 2\sqrt{6} + 2\sqrt{2}$.