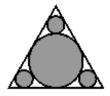23. The diagram shows four touching circles, each of which also touches the sides of an equilateral triangle with sides of length 3.

What is the area of the shaded region?

A $\dfrac{11\pi}{12}$ B π C $\dfrac{(4+\sqrt{3})\pi}{6}$ D $\dfrac{(3+\sqrt{3})\pi}{4}$ E $\dfrac{37\pi}{36}$

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23. **B** We note from the symmetry of the figure that the three small circles have the same radius. Let this be r and let the radius of the large circle be s. Let A, B, C, D, E be the points shown on the diagram.

By symmetry, $\angle DAE = 30°$.

Now $\dfrac{DE}{AD} = \sin 30° = \frac{1}{2}$ so AD has length $2s$. Similarly, AB has length $2r$.

Since $AD = AB + BC + CD$, the length of AD is also given by $2r + r + s$. Hence $2s = 3r + s$, i.e. $s = 3r$.

Also, $\dfrac{DE}{AE} = \dfrac{s}{s/2} = \tan 30° = \frac{1}{\sqrt{3}}$ so $s = \dfrac{3}{2\sqrt{3}}$. Hence $r = \dfrac{1}{2\sqrt{3}}$.

Thus the shaded area $= \pi s^2 + 3\pi r^2 = \pi \times \frac{9}{12} + 3\pi \times \frac{1}{12} = \pi$.