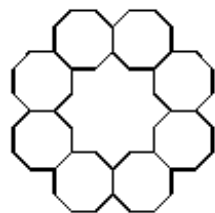




17. Eight identical regular octagons are placed edge to edge in a ring in such a way that a symmetrical star shape is formed by the interior edges. If each octagon has sides of length 1, what is the area of the star?



- A  $5 + 10\sqrt{2}$       B  $8\sqrt{2}$       C  $9 + 4\sqrt{2}$       D  $16 - 4\sqrt{2}$       E  $8 + 4\sqrt{2}$

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17. **E** Let  $A, B, C, D, E$  be five vertices of the star, as shown. Then  $AB = BC = CD = DE = 1$ . Each exterior angle of a regular octagon is  $360^\circ \div 8$ , that is  $45^\circ$ , so  $\angle CBD = \angle CDB = 45^\circ$ .

Hence  $\angle BCD$  is a right angle and we deduce from the symmetry of the figure that each interior angle of the star is either  $90^\circ$  or  $225^\circ$ . The length of  $BD$  is  $\sqrt{2}$ , so the area of the star is the area of a square of side  $2 + \sqrt{2}$  plus the area of four congruent triangles with sides 1, 1,  $\sqrt{2}$ .

The required area, therefore, is  $(2 + \sqrt{2})^2 + 4(\frac{1}{2} \times 1 \times 1)$ , that is  $6 + 4\sqrt{2} + 2$ , that is  $8 + 4\sqrt{2}$ .

*Possible alternative ending:* Dissect the star into 8 congruent kites such as  $OBCD$ . As for a rhombus, the area of a kite is half the product of its diagonals. In this case that is  $\frac{1}{2}OC \times BD = \frac{1}{2}(1 + \sqrt{2}) \times \sqrt{2} = \frac{1}{2}(\sqrt{2} + 2)$ . Required area is  $4(\sqrt{2} + 2)$ .

