Logic Puzzles
(From the Oxford MAT Tests)

For answers, see the MAT website

Specimen A, Question 6:

Alice, Bob and Charlie are well-known expert logicians.

(i) The King places a hat on each of their heads. Each of the logicians can see the others' hats, but not his or her own.

The King says "Each of your hats is either black or white, but you don't all have the same colour hat".

All four are honest, and all trust one another.

The King now asks Alice "Do you know what colour your hat is?".

Alice says "Yes, it's white".

What colour are the others' hats? [Hint: think about how Alice can deduce that her hat is white.]

(ii) The King now changes some of the hats, and again says "Each of your hats is either black or white, but you don't all have the same colour hat". He now asks Alice "Do you know what colour your hat is?".

Alice replies "No"

Can Bob work out what colour his hat is? Explain your answer. [Hint: what can Bob deduce from the fact that Alice can't tell what colour her hat is?]

(iii) The King now changes some of the hats, and then says "Each of your hats is either black or white. At least one of you has a white hat."

He now asks them all "Do you know what colour your hat is?". They all simultaneously reply "No".

What can you deduce about the colour of their hats? Explain your answer.

(iv) He again asks "Do you know what colour your hat is?" Alice says "No", but Bob and Charlie both say "Yes" (all three answer simultaneously).

What colour are their hats? Explain your answer.

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Specimen B, Question 6:

(i) Alice, Bob, Charlie and Dianne each make the following statements:

Alice:	I am telling the truth.
Bob:	Alice is telling the truth.
Charlie:	Bob is telling the truth.
Dianne:	Charlie is lying.

Only one of the 4 people is telling the truth. Which one? Explain your answer.

(ii)They now make the following statements:

Alice:	Bob is lying.
Bob:	Charlie is lying.
Charlie:	I like beer.
Dianne:	2+2=4.

Now two of the four people are telling the truth. Which two? Explain your answer.

(iii) They are now joined by Egbert. They each make the following statements:

Alice:	I like wine.
Bob:	Charlie is lying.
Charlie:	Alice is lying.
Dianne:	Alice likes beer.
Egbert:	Alice likes beer.

Now three of the five people are telling the truth. Which ones? Explain your answer.

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2007, Question 6:

Three people called Alf, Beth, and Gemma, sit together in the same room.

One of them always tells the truth.
One of them always tells a lie.
The other one tells truth or lies at random.

In each of the following situations, your task is determine how each person acts.

(i) Suppose that Alf says "I always tell lies" and Beth says "Yes, that's true, Alf always tells lies".

Who always tells the truth? Who always lies? Briefly explain your answer.

(ii) Suppose instead that Gemma says "Beth always tells the truth" and Beth says "That's wrong."

Who always tells the truth? Who always lies? Briefly explain your answer.

(iii) Suppose instead that Alf says "Beth is the one who behaves randomly" and Gemma says "Alf always lies". Then Beth says "You have heard enough to determine who always tells the truth".

Who always tells the truth? Who always lies? Briefly explain your answer.

Logic Puzzles
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2008, Question 6:

(i) A, B and C are three people. One of them is a *liar* who always tells lies, another is a *saint* who always tells the truth, and the third is a *switcher* who sometimes tells the truth and sometimes lies. They make the following statements:

A: I am the liar.
B: A is the liar.
C: I am not the liar.

Who is the liar among A, B and C? Briefly explain your answer.

(ii) P, Q and R are three more people, one a liar, one a saint, and the third a *contrarian* who tells a lie if he is the first to speak or if the preceding speaker told the truth, but otherwise tells the truth. They make the following statements:

P: Q is the liar.
Q: R is the liar.
R: P is the liar.

Who is the liar among P, Q and R? Briefly explain your answer.

(iii) X, Y and Z are three more people, one a liar, one a switcher and one a contrarian. They make the following statements:

X: Y is the liar.
Y: Z is the liar.
Z: X is the liar.
X: Y is the liar.
Y: X is the liar.

Who is the liar among X, Y and Z? Briefly explain your answer.

Logic Puzzles
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For answers, see (the MAT website)

2009, Question 6:

(i) Alice, Bob, and Charlie make the following statements:

Alice: Bob is lying.
Bob: Charlie is lying.
Charlie: $1 + 1 = 2$.

Who is telling the truth? Who is lying?. Explain your answer.

(ii) Now Alice, Bob, and Charlie make the following statements:

Alice: Bob is telling the truth.
Bob: Alice is telling the truth.
Charlie: Alice is lying.

What are the possible numbers of people telling the truth? Explain your answer.

(iii) They now make the following statements:

Alice: Bob and Charlie are both lying.
Bob: Alice is telling the truth or Charlie is lying (or both).
Charlie: Alice and Bob are both telling the truth.

Who is telling the truth and who is lying on this occasion? Explain your answer.

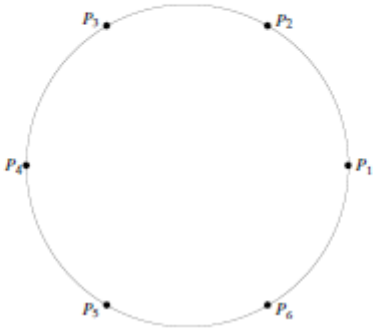
Logic Puzzles
(From the Oxford MAT Tests)

For answers, see (the MAT website)

2010, Question 6:

In the questions below, the people involved make statements about each other. Each person is either a *saint* (S) who always tells the truth or a *liar* (L) who always lies.

(i) Six people, $P_1, P_2, \ldots, P_6$ sit in order around a circular table with P_1 sitting to P_6's right, as shown in the diagram below.



(a) Suppose all six people say "the person directly opposite me is telling the truth". One possibility is that all six are lying. But, in total, how many different possibilities are there? Explain your reasoning.

(b) Suppose now that all six people say "the person to my left is lying". In how many different ways can this happen? Explain your reasoining.

(ii) Now n people $Q_1, Q_2, \ldots, Q_n$ sit in order around a circular table with Q_1 sitting to Q_n's right.

(a) Suppose that all n people make the statement "the person on my left is lying *and* the person on my right is telling the truth". Explain why everyone is lying.

(b) Suppose now that every person makes the statement "either the people to my left and right are both lying *or* both are telling the truth". If at least one person is lying, show that n is a multiple of three.

Logic Puzzles
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2011, Question 6:

Alice, Bob, Charlie and Diane are playing together when one of them breaks a precious vase. They all know who broke the vase. When questioned they make the following statements:

Alice:	It was Bob.
Bob:	It was Diane.
Charlie:	It was not me.
Diane:	What Bob says is wrong.

Each statement is either true or false.

(i) Explain why at least one of the four must be lying.

(ii) Explain why at least one of them must be telling the truth.

(iii) Let us suppose that exactly one of the four is lying, so the other three are telling the truth. Who is lying? Who did break the vase? Explain your answer.

(iv) Let us now suppose that exactly one of the four is telling the truth, so the other three are lying. Who is telling the truth? Who did break the vase? Explain your answer.

(v) Let us now suppose that two of the statements are true and two are false. List the people who might now have broken the vase. Justify your answers.

(vi) Hence show that if we don't know how many of the four statements are true, then any one of the four could have broken the vase.

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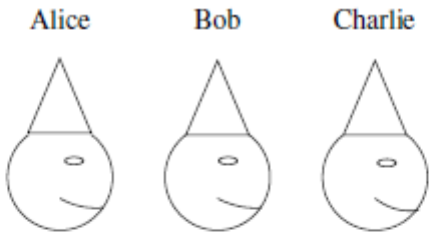
2012, Question 6:



Alice, Bob and Charlie are well-known expert logicians; they always tell the truth.

They are sat in a row, as illustrated above. In each of the scenarios below, their father puts a red or blue hat on each of their heads. Alice can see Bob's and Charlie's hats, but not her own; Bob can see only Charlie's hat; Charlie can see none of the hats. All three of them are aware of this arrangement.

(i) Their father puts a hat on each of their heads and says: "Each of your hats is either red or blue. At least one of you has a red hat." Alice then says "I know the colour of my hat." What colour is each person's hat? Explain your answer.

(ii) Their father puts a new hat on each of their heads and again says: "Each of your hats is either red or blue. At least one of you has a red hat." Alice then says "I don't know the colour of my hat." Bob then says "I don't know the colour of my hat." What colour is Charlie's hat? Explain your answer.

(iii) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. At least one of you has a red hat, and at least one of you has a blue hat." Alice says "I know the colour of my hat." Bob then says "Mine is red." What colour is each person's hat? Explain your answer.

(iv) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. At least one of you has a red hat, and at least one of you has a blue hat." Alice then says "I don't know the colour of my hat." Bob then says "My hat is red". What colour is Charlie's hat? Explain your answer.

(v) Their father puts a new hat on each of their heads and says: "Each of your hats is either red or blue. Two of you who are seated adjacently both have red hats." Alice then says "I don't know the colour of my hat." What colour is Charlie's hat? Explain your answer.

Logic Puzzles
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2013, Question 6:

Alice, Bob and Charlie are well-known expert logicians; they always tell the truth.

In each of the scenarios below, Charlie writes a whole number on Alice and Bob's foreheads. The difference between the two numbers is one: either Alice's number is one larger than Bob's, or Bob's number is one larger than Alice's. Each of Alice and Bob can see the number on the other's forehead, but can't see their own number.

(i) Charlie writes a number on Alice and Bob's foreheads, and says "Each of your numbers is at least 1. The difference between the numbers is 1."

Alice then says "I know my number."

Explain why Alice's number must be 2. What is Bob's number?

(ii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1. Alice's number is a prime." (A prime number is a number greater than 1 that is divisible only by 1 and itself.)

Alice then says "I don't know my number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.

(iii) Charlie now writes new numbers on their foreheads, and says "Each of your numbers is between 1 and 10 inclusive. The difference between the numbers is 1."

Alice then says "I don't know my number. Is my number a square number?"

Charlie then says "If I told you that, you would know your number."

Bob then says "I don't know my number."

What is Alice's number? Explain your answer.

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2014, Question 6:

Alice plays a game 5 times with her friends Sam and Pam. In each game Alice chooses two integers x and y with $1 \leqslant x \leqslant y$. She whispers the sum $x + y$ to Sam, and the product $x \times y$ to Pam, so that neither knows what the other was told. Sam and Pam then have to try to work out what the numbers x and y are. Sam and Pam are well known expert logicians.

(i) In the first game, Pam says "I know x and y."
What can we deduce about the values of x and y? Explain your answer.

(ii) In the second game, Pam says "I don't know what x and y are."
Sam then says "I know x and y."
Suppose the sum is 4. What are x and y? Explain your answer.

(iii) In the third game, Pam says "I don't know what x and y are."
Sam then says "I don't know what x and y are."
Pam then says "I now know x and y."
Suppose the product is 4. What are x and y? Explain your answer.

(iv) In the fourth game, Pam says "I don't know what x and y are."
Sam then says "I already knew that."
Pam then says "I now know x and y."
Suppose the product is 8. What are x and y? Explain your answer.

(v) Finally, in the fifth game, Pam says "I don't know what x and y are."
Sam then says "I don't know what x and y are."
Pam then says "I don't know what x and y are."
Sam then says "I now know x and y."
Suppose the sum is 5. What are x and y? Explain your answer.