Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MS04

Unit Statistics 4

Tuesday 24 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 The continuous random variable T has probability density function $\mathrm{f}(t)$, where

$$\mathrm{f}(t) = \begin{cases} 5\mathrm{e}^{-5t} & t \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Derive the cumulative distribution function of T.

[4 marks]

(b) Find the probability that $T > \mathrm{E}(T)$.

[1 mark]

(c) Find the value of the constant c such that $\mathrm{P}(T > c) = 0.05$.

[2 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE	Answer space for question 1

2 Victoria cycles to work every morning. She records the times taken, in minutes, to complete her journey for a random sample of 10 mornings. Her times are as follows.

$$22.6 \quad 20.9 \quad 25.8 \quad 24.3 \quad 26.3 \quad 21.9 \quad 23.2 \quad 22.7 \quad 21.3 \quad 22.8$$

(a) State a necessary assumption in order to construct a confidence interval for σ^2, the variance of Victoria's journey times.

[1 mark]

(b) Making the necessary assumption, construct a 98% confidence interval for σ^2.

[6 marks]

QUESTION PART REFERENCE

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3 The broadband speed, X Mbps, in rural villages may be assumed to be normally distributed with variance ${\sigma_X}^2$. The broadband speed, Y Mbps, in small towns may be assumed to be normally distributed with variance ${\sigma_Y}^2$.

The broadband speeds, x Mbps, in a random sample of 12 rural villages were as follows.

1.9 2.6 1.8 3.4 2.2 3.0 2.7 3.7 2.7 1.9 3.4 3.1

The broadband speeds, y Mbps, in a random sample of 9 small towns were as follows.

7.8 7.7 7.5 7.7 8.0 7.3 7.7 7.4 7.8

(a) Determine a 99% confidence interval for the variance ratio $\dfrac{{\sigma_X}^2}{{\sigma_Y}^2}$.

[7 marks]

(b) Hence comment on the suggestion that the broadband speed in rural villages is more variable than that in small towns.

[2 marks]

QUESTION PART REFERENCE

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4 An ergonomist is investigating the effect of training on the speed with which workers in a factory can assemble a particular product. The ergonomist selects a random sample of 8 workers who have not received the training and a random sample of 6 workers who have received the training. The ergonomist records the time taken, in minutes, for each of these selected workers to assemble the product. The results are shown in the table below.

Untrained	10.4	8.9	10.1	9.0	9.4	9.6	10.0	10.2
Trained	9.0	8.3	9.5	8.0	9.2	8.2		

(a) State **two** necessary assumptions in order to test the hypothesis that the mean time taken by the untrained workers is the same as the mean time taken by the trained workers.

[2 marks]

(b) Given that all the necessary assumptions are valid, test the hypothesis in part **(a)** using the 2% level of significance.

[10 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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5 Coloured plastic clips are sold in packets of 12 clips. It is suggested that the number of blue clips in a packet can be modelled by a binomial distribution.

In order to investigate this suggestion, 100 packets of clips are randomly chosen. The number of blue clips in each packet is counted with the following summarised results.

Number of blue clips	0	1	2	3	4	5	6	$\geqslant 7$
Number of packets	0	6	14	28	27	16	9	0

(a) Show that an estimate of p, the probability that a randomly chosen clip is blue, is 0.3. **[2 marks]**

(b) Test, at the 10% level of significance, whether a binomial distribution is an appropriate model for the number of blue clips in a packet. **[10 marks]**

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6 Two independent random samples of observations, of sizes n_1 and n_2, are made of a random variable X, which has mean μ and variance σ^2. The sample means are denoted by $\overline{X}_1$ and $\overline{X}_2$ respectively.

(a) Show that $T = k\overline{X}_1 + (1-k)\overline{X}_2$ is an unbiased estimator of μ.

[2 marks]

(b) Show that V, the variance of T, is given by

$$V = k^2 \frac{\sigma^2}{n_1} + (1-k)^2 \frac{\sigma^2}{n_2}$$

[2 marks]

(c) Find the value of k for which $\frac{\mathrm{d}V}{\mathrm{d}k} = 0$.

[3 marks]

(d) For the value of k found in part **(c)**:

(i) find an expression for T;

[2 marks]

(ii) interpret the expression found in part **(d)(i)**;

[1 mark]

(iii) find $\frac{\mathrm{d}^2V}{\mathrm{d}k^2}$ and hence comment on what you can deduce about V.

[2 marks]

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7 (a) The random variable X has a geometric distribution with parameter p.

(i) Prove, from first principles, that $\mathrm{E}(X^2) = \frac{1}{p} + \frac{2(1-p)}{p^2}$.

[4 marks]

(ii) Hence, given that $\mathrm{E}(X) = \frac{1}{p}$, deduce that $\mathrm{Var}(X) = \frac{(1-p)}{p^2}$.

[1 mark]

(iii) Given that $p = \frac{1}{2}$, calculate $\mathrm{P}(X > \mathrm{Var}(X))$.

[3 marks]

(b) As part of their archery practice, Robin and William are playing a game consisting of a number of rounds. For each round of the game, they each shoot one arrow at the gold inner circle of a target. The probability that Robin hits the gold with any one arrow is $\frac{1}{5}$, independently of all previous shots. The probability that William hits the gold with any one arrow is $\frac{1}{6}$, independently of all previous shots. In each round, Robin shoots first.

If, in a round, they both hit the gold, then the game is drawn.

If, in a round, Robin hits the gold and then William misses the gold, then Robin wins the game.

If, in a round, Robin misses the gold and then William hits the gold, then William wins the game.

If, in a round, they both miss the gold, then the game continues to the next round.

Find the probability that:

(i) the game is drawn after no more than three rounds have been completed;

[3 marks]

(ii) the game is drawn;

[2 marks]

(iii) Robin wins the game.

[3 marks]

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