Version: 1.0 0608

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}\left(-\frac{1}{3}\right) = 27\times\left(-\frac{1}{3}\right)^3 - 9\times\left(-\frac{1}{3}\right) + 2$	M1		Use of $\pm\frac{1}{3}$ or complete division with integer remainder M1
	$= -1 + 3 + 2 = 4$	A1	2	remainder = 4 indicated A1
(b)(i)	$\mathrm{f}\left(-\frac{2}{3}\right) = -8 + 6 + 2 = 0$	B1	1	AG
(b)(ii)	$\mathrm{f}(x) = (3x+2)(ax^2 + bx + c)$	B1		$(3x+2)$ or $\left(x+\frac{2}{3}\right)$ is a factor PI
	$a = 9$ $c = 1$ x^2 term $3b + 2a = 0$ or x term $3c + 2b = -9$	M1		quadratic factor; find coefficients; 2 correct equate coefficients and solve for b
	$b = -6$ or (could be shown as) $9x^2 - 6x + 1$	A1		correct quadratic factor or a, b, and c correct or use division or factor theorem to seek another factor (see alternative methods at end of scheme)
	$\mathrm{f}(x) = (3x+2)(3x-1)(3x-1)$	A1	4	SC (see alternative methods at end of scheme)
(b)(iii)	$9x^2 + 3x - 2 = (3x-1)(3x+2)$	M1		factorise denominator correctly or complete division
	$\dfrac{27x^3 - 9x + 2}{9x^2 + 3x - 2} = 3x - 1$	A1	2	simplified result indicated
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{dx}{dt}=4 \quad \frac{dy}{dt}=-\frac{1}{2t^2}$	M1 A1		differentiate. 4; at^{-2} seen both derivatives correct
	$\frac{dy}{dx}=-\frac{1}{2t^2}\times\frac{1}{4}$	M1		use chain rule candidates' $\frac{dy}{dt} \Big/ \frac{dx}{dt}$
	$t=\frac{1}{2} \quad \frac{dy}{dx}=-\frac{1}{2}$	A1	4	CSO
(b)	gradient of normal = 2 $(x,y)=(5,0) \quad \frac{y}{x-5}=2$	B1F M1 A1F	3	F if gradient $\neq \pm 1$ calculate and use (x, y) on normal F on gradient of normal ACF
(c)	$x-3=4t \quad$ or $\quad y+1=\frac{1}{2t}$	B1		or $t=\frac{x-3}{4}$ or $\frac{1}{t}=2(y+1)$
	$(x-3)(y+1)=2$	M1 A1	3	eliminate t; allow one error accept $y=\frac{1}{\frac{2(x-3)}{4}}-1$ ACF SC allow marks for part (c) if done in part (a)
	Total		**10**	
3(a)	$\sin(x+2x)=\sin x\cos 2x+\cos x\sin 2x$	M1		
	$=\sin x(1-2\sin^2 x)+\cos x(2\sin x\cos x)$	B1B1		double angles; ACF ISW condone missing x
	$=\sin x(1-2\sin^2 x)+2\sin x(1-\sin^2 x)$	A1		all in $\sin x$, correct expression
	$=3\sin x-2\sin^3 x-2\sin^3 x$ $=3\sin x-4\sin^3 x$	A1	5	CSO AG
(b)	$\sin^3 x=a\sin x+b\sin 3x$	M1		attempt to solve for $\sin^3 x$ where $a\neq 0$ and $b\neq 0$
	$\int\sin^3 x\,dx=-a\cos x-\frac{b}{3}\cos 3x$	A1F		either integral correct F on a, b
	$\int\sin^3 x\ dx=\frac{1}{4}\left(-3\cos x+\frac{1}{3}\cos 3x\right)\ (+C$	A1	3	CAO alternative method by parts (see end of mark scheme)
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$(1-x)^{\frac{1}{4}} = 1+\frac{1}{4}(-x)+\frac{1}{2}\times\frac{1}{4}\left(-\frac{3}{4}\right)(-x)^2$	M1		$1 \pm \frac{1}{4}x + kx^2$
	$= 1-\frac{1}{4}x-\frac{3}{32}x^2$	A1	2	equivalent fractions or decimals
(a)(ii)	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}}\left(1-\frac{16}{81}x\right)^{\frac{1}{4}}$	B1		
	$= k\left(1-\frac{1}{4}\times\frac{16}{81}x-\frac{3}{32}\left(\frac{16}{81}x\right)^2\right)$	M1		x replaced by $\frac{16}{81}x$
				or start binomial again condone one error (missing bracket; x or x^2; sign error)
	= 3( )			
	$= 3-\frac{4}{27}x-\frac{8}{729}x^2$	A1	3	CSO AG
				use of $(a+bx)^n$ ignoring hence (see end of mark scheme)
(b)	$3-\frac{4}{27}\times\frac{1}{16}-\frac{8}{729}\left(\frac{1}{16}\right)^2$	M1		use $x=\frac{1}{16}$
	= 2.9906979	A1	2	seven decimal places only
	Total		**7**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\cos\alpha = \frac{3}{5}$	B1	1	ACF
(a)(ii)	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	M1		
	$= \frac{3}{5}\cos\beta + \frac{4}{5}\sin\beta$	A1	2	ACF
(a)(iii)	$\sin\beta = \frac{12}{13}$	B1		
	$\cos(\alpha - \beta) = \frac{63}{65}$	B1	2	$\frac{63}{65}$ NMS B1B1
(b)(i)	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	M1		
	$2\tan x = 1 - \tan^2 x$			
	$\tan^2 x + 2\tan x - 1 = 0$	A1	2	CSO AG
(b)(ii)	$\tan x = \frac{-2 \pm \sqrt{4+4}}{2}$	M1		must solve quadratic equation by formula or by completing the square condone one slip
	$= -1 \pm \sqrt{2}$	A1		$\pm\sqrt{2}$ required
	$2x = 45° \Rightarrow x = 22\frac{1}{2}°$ is acute			
	$\Rightarrow \tan 22\frac{1}{2}° = \sqrt{2} - 1$	E1	3	explain selection of positive root
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{2}{(x^2-1)} = \frac{A}{x-1} + \frac{B}{x+1}$			
	$2 = A(x+1) + B(x-1)$	M1		
	$x = 1 \qquad x = -1$	m1		use two values of x or equate coefficients and solve $A + B = 0$ and $A - B = 2$
	$A = 1 \qquad B = -1$	A1	3	both A and B
(b)	$\int \frac{2}{x^2-1}\,dx = p\ln(x-1) + q\ln(x+1)$	M1		ln integrals
	$= \ln(x-1) - \ln(x+1)$	A1F	2	F on A and B condone missing brackets
(c)	$\int \frac{dy}{y} = \int \frac{2}{3(x^2-1)}\,dx$	M1		separate and attempt to integrate on one side
	$\ln y = \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right) \quad (+C)$	A1 A1F		left hand side F from part (b) on right hand side
	$(3,1) \qquad \ln 1 = \frac{1}{3}(\ln 2 - \ln 4) + C$	m1		use (3, 1) to attempt to find a constant
	$3\ln y = \ln(x-1) - \ln(x+1) - (\ln 2 - \ln 4)$			
	$3\ln y = \left(\ln\left(\frac{x-1}{x+1}\right) + \ln 2\right)$			
	$\ln y^3 = \ln\left(\frac{2(x-1)}{x+1}\right)$			
	$y^3 = \frac{2(x-1)}{x+1}$	A1	5	CSO AG
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$AB^2 = (5-3)^2 + (3--2)^2 + (0-1)^2$	M1		use $\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ in sum of squares of components allow one slip in difference
	$AB = \sqrt{30}$	A1	2	accept 5.5 or better
(b)	$\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} = 2+3=5$	M1		$\pm\overrightarrow{AB}\bullet$ direction l evaluated condone one component error
		A1		5 or – 5
	$\cos\theta = \dfrac{5}{\sqrt{30}\sqrt{10}}$	B1F M1		F on either of candidates' vectors use $\lvert a\rvert\lvert b\rvert\cos\theta = a\bullet b$; values needed
	$\theta = 73°$	A1	5	CAO (condone 73.2, 73.22 or 73.22…)
(c)	$\overrightarrow{AC} = \begin{bmatrix} 5+\lambda \\ 3 \\ -3\lambda \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+\lambda \\ 5 \\ -1-3\lambda \end{bmatrix}$	M1		for $\overrightarrow{OC}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OC}$ with $\overrightarrow{OC}$ in terms of λ condone one component error
		A1		
	$(2+\lambda)^2 + 5^2 + (-1-3\lambda)^2 = 30$ $10\lambda^2 + 10\lambda = 0$	m1		
	$(\lambda = 0 \text{ or})\ \lambda = -1$ $(\lambda = 0 \Rightarrow (5,3,0)$ is B)	A1		
	$\lambda = -1 \Rightarrow C$ is $(4,3,3)$	A1	5	condone $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$p\dfrac{\mathrm{d}x}{\mathrm{d}t}=q$	M1		where p and q are functions
	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=-kx$	A1	2	in any correct combination
(a)(ii)	$-500=-k\,20000$ or $500=k\,20000$	M1		condone sign error or missing 0 k can be on either side of the equation
	$k=\dfrac{5}{200}\quad(=0.025)$	A1	2	CSO both (a)(i) and (a)(ii)
(b)(i)	A = 1300	B1	1	
(b)(ii)	$100>A\mathrm{e}^{-0.05\,t}$	M1		condone = for >; condone 99 for 100
	$\ln\left(\dfrac{100}{A}\right)>-0.05\,t$	m1		take logs correctly condone 0.5
	$t>51.3$	A1		or by trial and improvement (see end of mark scheme)
	population first exceeds 1900 in 2059	A1F	4	F if M1 m1 earned and t>0 following A
	Total		**9**	
	TOTAL		**75**	

MPC4 (cont)
Alternative methods permitted in the mark scheme

Q	Solution	Marks	Total	Comments
1(b)(ii)	**ALTERNATIVE METHOD 1**			
	$(3x+2)$ is a factor	B1		PI
	use factor theorem	M1		use factor theorem or algebraic division to find another factor
	$\text{f}\left(\frac{1}{3}\right)=0 \Rightarrow (3x-1)$ is a factor			
	$\text{f}(x)=(3x+2)(3x-1)(ax+b)$	A1		
	$\text{f}(x)=(3x+2)(3x-1)(3x-1)$	A1	4	
	ALTERNATIVE METHOD 2			
	$(3x+2)$ is a factor	B1		PI by division
	divide $27x^3-9x+2$ by $(3x+2)$	M1		complete division to ax^2+bx+c
	$9x^2-6x+1$	A1		
	$\text{f}(x)=(3x+2)(3x-1)(3x-1)$	A1	4	
1(b)(ii)	**SPECIAL CASE**			
	$(3x+2)(3x-1)(ax+b)$		2	
2(a)	$y=\frac{2}{x-3}-1$ and differentiate	M1		differentiate expression in y and x
	$\frac{\text{d}y}{\text{d}x}=\frac{-2}{(x-3)^2}$	A1		correct
	$x=5$			
	$\frac{\text{d}y}{\text{d}x}=\frac{-2}{(5-3)^2}$	m1		find and therefore use x (and y)
	$\frac{\text{d}y}{\text{d}x}=-\frac{1}{2}$	A1	4	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(b)	**ALTERNATIVE METHOD 1**			
	$\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx$ = $-\sin^2 x\cos x - \int -2\cos x\sin x\cos x\,dx$	M1		identify parts and attempt to integrate
	$= -\sin^2 x\cos x - \frac{2}{3}\cos^3 x \quad (+C)$	A2	3	
	ALTERNATIVE METHOD 2			
	$\int \sin^3 x\,dx = \int \sin^2 x\,d(-\cos x)$	M1		condone sign error
	$= \int -(1-\cos^2 x)\,d(\cos x)$			
	$= -\cos x + \frac{1}{3}\cos^3 x \quad (+C)$	A2	3	
	ALTERNATIVE METHOD 3			
	$\int \sin x \sin^2 x\,dx$			
	$\int \sin x(1-\cos^2 x)\,dx$	M1		this form and attempt to integrate
	$= -\cos x + \frac{1}{3}\cos^3 x \quad (+C)$	A2	3	
4(a)(ii)				using $(a+bx)^n$ from FB
	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}} + \frac{1}{4}81^{-\frac{3}{4}}(-16x) + \frac{1}{4}\left(-\frac{3}{4}\right)\frac{1}{2}81^{-\frac{7}{4}}(-16x)^2$	M1 A1		condone one error
	$= \left(3 - \frac{4}{27}x - \frac{8}{729}x^2\right)$	A1	3	CSO completely correct
8(b)(ii)	$t = 51 \to 101.5$ $t = 52 \to 96.6$ $\Rightarrow 51 < t < 52$ population first exceeds 1900 in 2059	M1 A3	 4	t = 51 or t = 52 considered CAO