General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 15 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
• an 8-page answer book • the **blue** AQA booklet of formulae and statistical tables You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The curve $y = x^3 - x - 7$ intersects the x-axis at the point where $x = \alpha$.

(a) Show that α lies between 2.0 and 2.1. *(2 marks)*

(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 7}$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

2 (a) Find $\dfrac{dy}{dx}$ when $y = (3x - 1)^{10}$. *(2 marks)*

(b) Use the substitution $u = 2x + 1$ to find $\int x(2x + 1)^8 \, dx$, giving your answer in terms of x. *(4 marks)*

3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3 \sec x + 9$ can be written as

$$\sec^2 x - 3 \sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3 \sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x - 4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

5 (a) A curve has equation $y = e^{2x} - 10e^x + 12x$.

(i) Find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x-coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^x + 6 = 0$$ *(1 mark)*

(ii) By using the substitution $z = e^x$, or otherwise, show that the x-coordinates of P and Q are $\ln 2$ and $\ln 3$. *(3 marks)*

(iii) Find the y-coordinates of P and Q, giving each of your answers in the form $m + 12 \ln n$, where m and n are integers. *(3 marks)*

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. *(3 marks)*

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\displaystyle\int_1^5 \ln x \, dx$, giving your answer to three significant figures. *(3 marks)*

(b) (i) Given that $y = x \ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Hence, or otherwise, find $\displaystyle\int \ln x \, dx$. *(2 marks)*

(iii) Find the exact value of $\displaystyle\int_1^5 \ln x \, dx$. *(2 marks)*

7 (a) Given that $z = \dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{dz}{dx} = \sec^2 x$. *(3 marks)*

(b) Sketch the curve with equation $y = \sec x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. *(2 marks)*

(c) The region R is bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(3 marks)*

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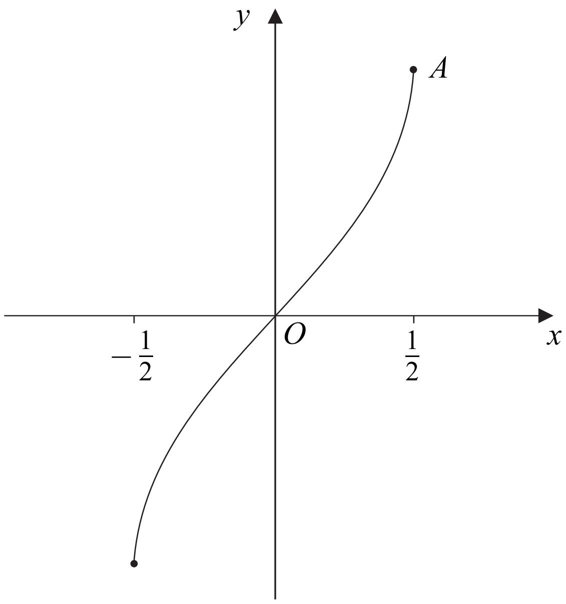
8 A function f is defined by $\mathrm{f}(x) = 2\mathrm{e}^{3x} - 1$ for all real values of x.

(a) Find the range of f. *(2 marks)*

(b) Show that $\mathrm{f}^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+1}{2}\right)$. *(3 marks)*

(c) Find the gradient of the curve $y = \mathrm{f}^{-1}(x)$ when $x = 0$. *(4 marks)*

9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.



(a) Find the y-coordinate of the point A, where $x = \frac{1}{2}$. *(1 mark)*

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2}\sin y$. *(1 mark)*

(ii) Given that $x = \frac{1}{2}\sin y$, find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. *(1 mark)*

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{1 - 4x^2}}$$ *(4 marks)*

END OF QUESTIONS