Version : 1.0 0109

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

$\sqrt{}$ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

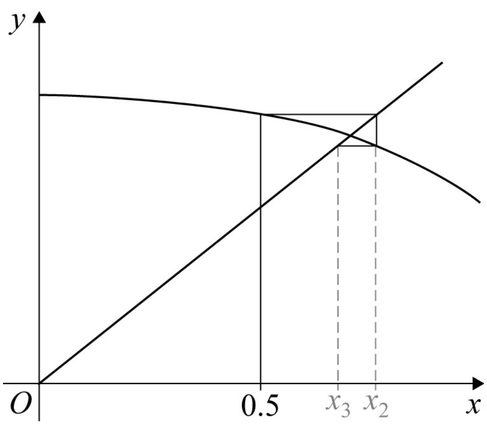
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1				
	x y 1 0.5	B1		x values and no extra values
	3 0.366(0) 5 0.309(0) 7 0.274(3) 9 0.25	B1		4+ correct y values or $\frac{1}{1+\sqrt{3}}$ etc
	$\int =$ $\frac{1}{3}\times 2\times\left[\begin{array}{l}(0.5+0.25)+\\ 4(0.3660+0.2743)+2(0.3090)\end{array}\right]$	M1		Correct application of Simpson's rule for their x values (x odd)
	$= 2.62$	A1	4	CSO must be 3sf
	Total		**4**	
2	$V=(\pi)\int y^2\,\mathrm{d}x$			
	$=(\pi)\int(x-2)^5\,\mathrm{d}x$	M1		
	$=(\pi)\left[\frac{(x-2)^6}{6}\right]_3^4$	A1		limits not required
	$=(\pi)\left(\frac{2^6}{6}-\frac{1}{6}\right)$	m1		correct substitution into $(\pi)k(x-2)^6$
	$=10.5\pi$	A1	4	allow equivalent fraction $\left(\frac{63}{6}\pi \text{ etc}\right)$ (AWRT 10.5 or 10.5π m1, A0)
	Total		**4**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{f}(x) = x^3 + 5x - 4$ $\mathrm{f}(0.5) = -1.375$ $\mathrm{f}(1) = 2$	M1		Condone $\mathrm{f}(0.5)$ rounding to -1.4
	Change of sign $\therefore 0.5 < \alpha < 1$	A1	2	Both statements needed
(b)	$x^3 + 5x - 4 = 0$ $5x = 4 - x^3$			Must be seen
	$x = \frac{1}{5}(4 - x^3)$	B1	1	AG
(c)	$x_1 = 0.5$ $(x_2 = 0.775)\ \left(= {}^{31}\!/\!_{40}\right)$	M1		For x_2 or $x_3 = (2\text{ sf})$
	$x_3 = 0.707$	A1	2	
(d)		M1 A1	 2	From 0.5 vertical to curve then horizontal to line CAO
	Total		**7**	

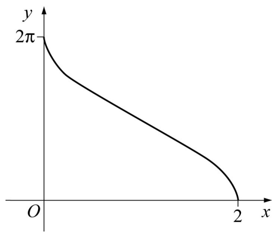
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\sec x = \frac{3}{2}$			
	$\cos x = \frac{2}{3}$			
	$x = 48, 312$	B1		1 correct
	(Condone answers rounding to)	B1	2	2 correct and no extras in interval
(b)	$2\tan^2 x = 10 - 5\sec x$			
	$2\left(\sec^2 x - 1\right) = 10 - 5\sec x$	M1		Use of trig identity correctly
	$2\sec^2 x + 5\sec x - 12 (= 0)$	A1		
	$(2\sec x - 3)(\sec x + 4)(= 0)$	m1		Attempt to solve or factorise 1 slip using formula
	$\left.\begin{array}{l}\sec x = \frac{3}{2},\ -4 \\ \cos x = \frac{2}{3}, -\frac{1}{4}\end{array}\right\}$ either of these	A1		
	$x = 48, 312, 104, 256$	B1		AWRT 3 correct condone 105 or 255
		B1	6	All correct and no extras in interval
	Alternative:			
	$\left.\begin{array}{l}\frac{2\sin^2 x}{\cos^2 x} = 10 - \frac{5}{\cos x} \\ 2\sin^2 x = 10\cos^2 x - 5\cos x \\ 2 - 2\cos^2 x = 10\cos^2 x - 5\cos x\end{array}\right\}$	(M1) (A1)		
	$12\cos^2 x - 5\cos x - 2 = 0$			
	then rest of scheme as above			
	Total		**8**	
5(a)	$\mathrm{f}(x) \le 2, \quad \mathrm{f} \le 2, \quad y \le 2$	B2	2	$\left.\begin{array}{l}\le 2, \mathrm{f}(x) < 2, x \le 2 \\ y < 2,\ \mathrm{f} < 2\end{array}\right\}$ B1
(b)	$\mathrm{f}(x)$ is not one to one	E1	1	Allow many to one or numerical example
(c)(i)	$\mathrm{fg}(x) = 2 - \left(\frac{1}{x-4}\right)^4$	B1	1	
(ii)	$2 - \left(\frac{1}{x-4}\right)^4 = -14$			
	$16 = \left(\frac{1}{x-4}\right)^4$			
	$\left.\begin{array}{l}(x-4)^4 = \frac{1}{16} \\ x - 4 = \pm\frac{1}{2}\end{array}\right\}$	M1 M1		Correct handling of fourth root Must have $\pm$ Correct handling of reciprocal
	$x = 4\frac{1}{2}, 3\frac{1}{2}$	A1	3	
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y = \mathrm{e}^{2x}\left(x^2 - 4x - 2\right)$			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}\left(2x-4\right)$	M1		Product rule; allow 1 slip
	$+\left(x^2-4x-2\right)2\mathrm{e}^{2x}$	A1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}\left(2x-4+2x^2-8x-4\right)$	M1		Factorising $\mathrm{e}^{2x}\ \left(ax^2 + 6x + 0\right)$
	$\mathrm{e}^{2x}\left(2x^2-6x-8\right)$	A1		or $x^2 - 3x - 4 = 0$
	$\mathrm{e}^{2x} \neq 0$			
	$\left(x-4\right)\left(x+1\right) = 0$	m1		Solving 3 term quadratic Dependent on both M marks
	$x = 4, -1$	A1	6	And no extras eg $x = 0$
(b)(i)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{2x}.2 + \left(2x-4\right)2\mathrm{e}^{2x}$	M1		Product rule from their $\frac{\mathrm{d}y}{\mathrm{d}x}$ in form
	$+\left(x^2-4x-2\right)4\mathrm{e}^{2x} + 2\mathrm{e}^{2x}\left(2x-4\right)$	A1	2	e^{2x} (quadratic) $\quad \mathrm{e}^{2x}\left(4x^2 - 8x - 22\right)$
	Or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{2x}\left(4x-6\right) + \left(2x^2-6x-8\right)2\mathrm{e}^{2x}$	M1 A1		
(ii)	$x = 4 : y'' = \mathrm{e}^{8}\left(10\right) > 0 \therefore$ MIN	M1		Their 2 x's in their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ only of form e^{2x} (quadratic)
	$x = -1 : y'' = \mathrm{e}^{-2}\left(-10\right) < 0 \therefore$ MAX	A1	2	CSO Both correct Allow values either side of y or y'
	Total		**10**	
7(a)	$3\mathrm{e}^{x} = 4$			
	$\mathrm{e}^{x} = \frac{4}{3}$	M1		
	$x = \ln\frac{4}{3}$	A1	2	
(b)(i)	$3\mathrm{e}^{x} + 20\mathrm{e}^{-x} = 19$			
	$3y + \frac{20}{y} = 19$ or $3\mathrm{e}^{2x} + 20 = 19\mathrm{e}^{x}$			
	$3y^2 - 19y + 20 = 0$	B1	1	AG
(ii)	$\left(3y-4\right)\left(y-5\right) = 0$			
	$y = \frac{4}{3}, 5$	B1		
	$\therefore x = \ln\frac{4}{3}, \ln 5$	M1 A1	3	ln (their + ve y's)
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$P\,(-1,\,\pi)$	B1		Condone $(\,-1,\,180°)$
	$Q\,(1,\,0)$	B1	2	
(b)	Translate	E1		
	$\begin{bmatrix}1\\0\end{bmatrix}$	B1		or equivalent in words
	Stretch SF 2 $/\!\!/$ y-axis	M1 A1	 4	Stretch + one other correct all correct
(c)		B1 B1	 2	Correct shape in 1st quadrant 2π and 2 marked correctly
(d)(i)	$\frac{y}{2}=\cos^{-1}(x-1)$	M1		
	$\cos\left(\frac{y}{2}\right)=x-1$			
	$x=\cos\left(\frac{y}{2}\right)+1$	A1	2	
(ii)	$-\frac{1}{2}\sin\left(\frac{y}{2}\right)$	M1 A1		$k\ \sin\ (...)$ $\frac{\mathrm{d}x}{\mathrm{d}y}$ correct
	At $y=2,\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)-\frac{1}{2}\sin 1$	A1	3	Condone AWRT –0.42
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$y=\frac{4x}{4x-3}$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(4x-3).4-4x(4)}{(4x-3)^2}$	M1		Must use quotient rule Condone one slip
	$=\frac{-12}{(4x-3)^2}$	A1	2	$k=-12$
(b)(i)	$y=x\ln(4x-3)$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x.4}{4x-3}+\ln(4x-3)$	M1		$\frac{\mathrm{f}(x)}{4x-3}+g(x)$ 'f(x)' may be constant
		m1		$\frac{kx}{4x-3}+\ln(4x-3)$
		A1	3	
(ii)	$x=1 \quad y=0$	B1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=4$	M1		Sub $x = 1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\therefore y=4(x-1)$ any correct form	A1	3	CSO Must have full marks in (b)(i)
(c)(i)	$u=4x-3$			
	$\mathrm{d}u=4\,\mathrm{d}x$	M1		
	$\int\frac{4x}{4x-3}\,\mathrm{d}x=\int\frac{u+3}{u}\,\frac{\mathrm{d}u}{4}$	A1		Or $\int\frac{4x}{4x-3}\mathrm{d}x=\int\left(1+\frac{3}{4x-3}\right)\mathrm{d}x$
	$=\left(\frac{1}{4}\right)\int\left(1+\frac{3}{u}\right)(\mathrm{d}u)$	m1		$=\int\left(1+\frac{3}{u}\right)\mathrm{d}u$ etc
	$=\frac{1}{4}(u+3\ln u)$			
	$=\frac{1}{4}\left[(4x-3)+3\ln(4x-3)\right](+c)$	A1	4	CSO Condone missing du
(ii)	$\int\ln(4x-3)\,\mathrm{d}x$			
	$u=\ln(4x-3) \quad \frac{\mathrm{d}v}{\mathrm{d}x}=1$	M1		In correct direction
	$\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{4}{4x-3} \quad v=x$			
	$\int=x\ln(4x-3)-\int\frac{4x}{4x-3}\,\mathrm{d}x$	A1		
	$=x\ln(4x-3)-\frac{1}{4}\left[(4x-3)+3\ln(4x-3)\right]$ $(+c)$	m1 A1	4	$x\ln(4x-3)-$ their (c)(i)
	Total		**16**	
	TOTAL		**75**	