AQA

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MPC3

Unit Pure Core 3

Thursday 6 June 2013   9.00 am to 10.30 am

For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
• Do not write outside the box around each page.
• Show all necessary working; otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.
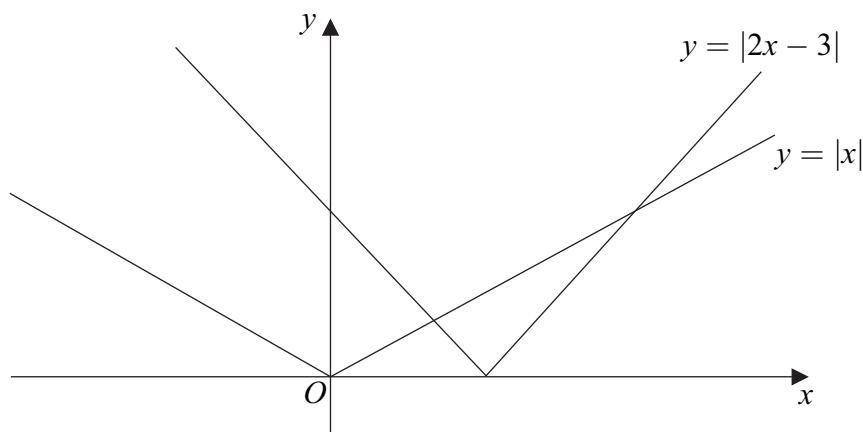• You do not necessarily need to use all the space provided.

MPC3

Answer **all** questions.

Answer each question in the space provided for that question.

1 The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.

(a) Find the x-coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$. *(3 marks)*

(b) Hence, or otherwise, solve the inequality

$$|2x - 3| \geqslant |x|$$

(2 marks)

QUESTION PART REFERENCE	**Answer space for question 1**

QUESTION PART REFERENCE	**Answer space for question 1**

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2 (a)   Given that $y = x^4 \tan 2x$, find $\dfrac{dy}{dx}$.   *(3 marks)*

(b)   Find the gradient of the curve with equation $y = \dfrac{x^2}{x-1}$ at the point where $x = 3$.

(3 marks)

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	Answer space for question 2

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3 (a) The equation $e^{-x} - 2 + \sqrt{x} = 0$ has a single root, α.

Show that α lies between 3 and 4. *(2 marks)*

(b) Use the recurrence relation $x_{n+1} = (2 - e^{-x_n})^2$, with $x_1 = 3.5$, to find x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(c) The diagram on the opposite page shows parts of the graphs of $y = (2 - e^{-x})^2$ and $y = x$, and a position of x_1.

On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

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(c)
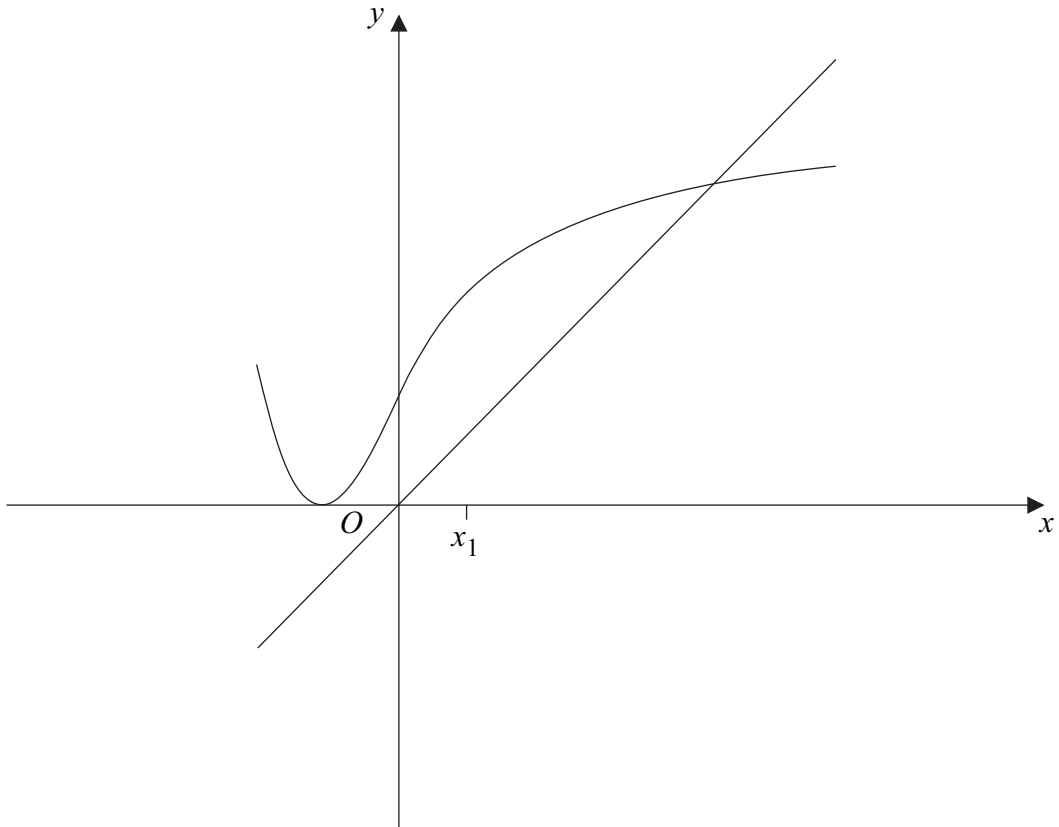
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4 By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures.

(7 marks)

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5   The diagram shows a sketch of the graph of $y = \sqrt{27 + x^3}$.
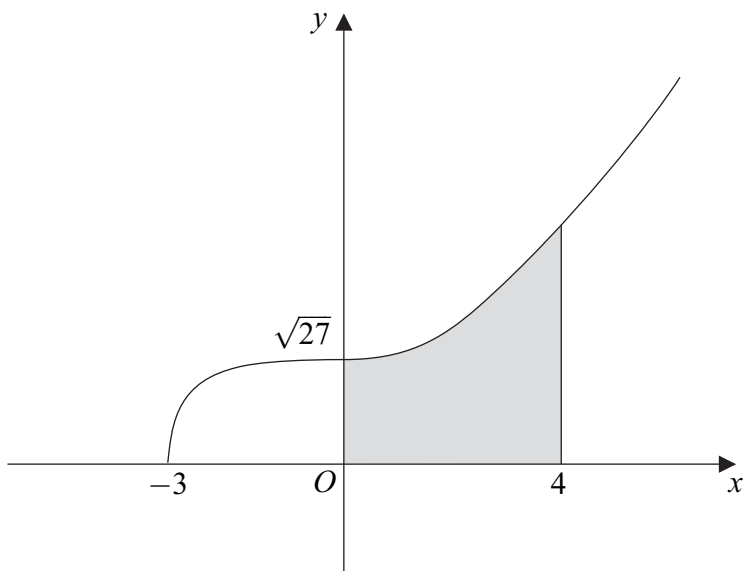
(a)   The area of the shaded region, bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, is given by $\int_0^4 \sqrt{27 + x^3}\, dx$.

Use the mid-ordinate rule with **five** strips to find an estimate for this area. Give your answer to three significant figures.   *(4 marks)*

(b)   With the aid of a diagram, explain whether the mid-ordinate rule applied in part **(a)** gives an estimate which is smaller than or greater than the area of the shaded region.   *(2 marks)*

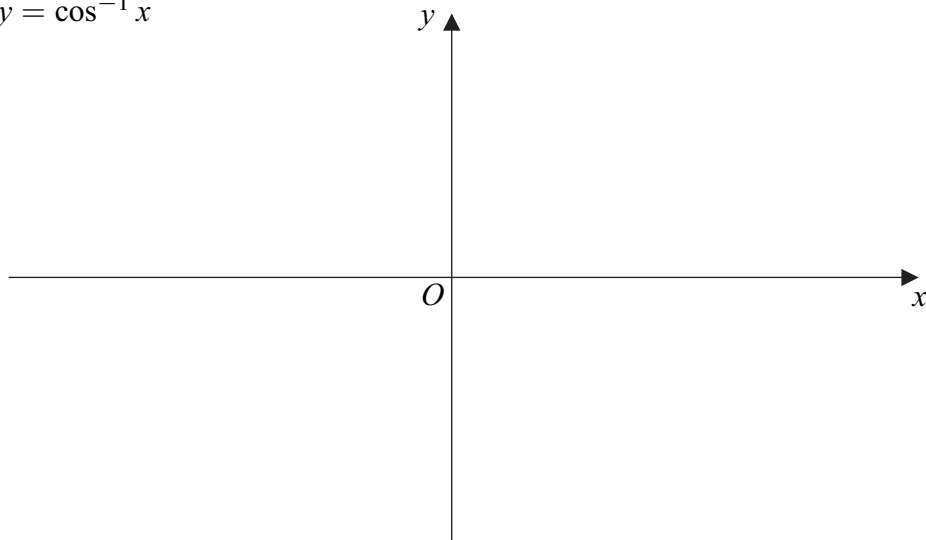| QUESTION PART REFERENCE	**Answer space for question 5**

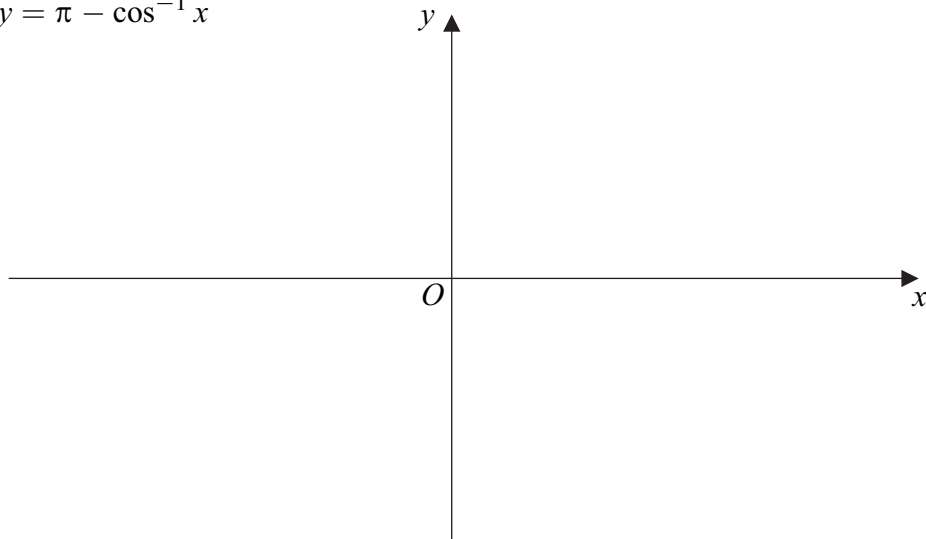| QUESTION PART REFERENCE	Answer space for question 5

6 (a) Sketch the graph of $y = \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

(b) Sketch the graph of $y = \pi - \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 6

(a)

$y = \cos^{-1} x$
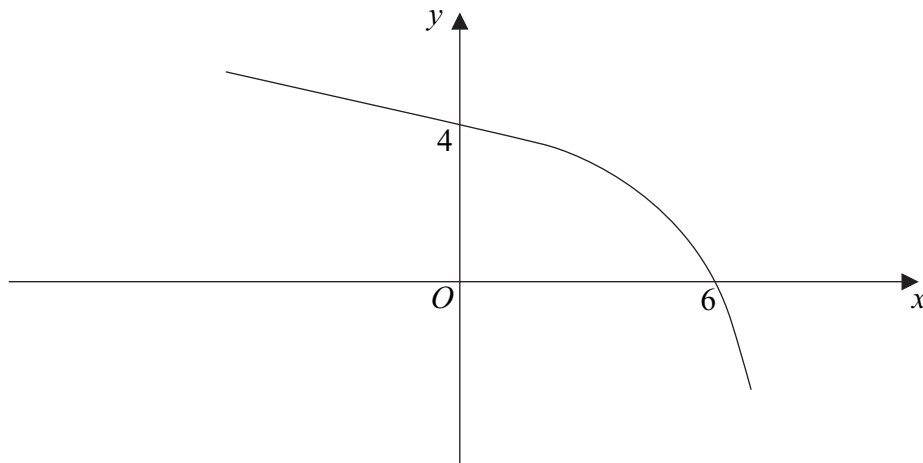

(b)

$y = \pi - \cos^{-1} x$

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7 The diagram shows a sketch of the curve with equation $y = f(x)$.

(a) On **Figure 1**, below, sketch the curve with equation $y = -f(3x)$, indicating the values where the curve cuts the coordinate axes. *(2 marks)*

(b) On **Figure 2**, on the opposite page, sketch the curve with equation $y = f(|x|)$, indicating the values where the curve cuts the coordinate axes. *(3 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f\left(-\frac{1}{2}x\right)$. *(4 marks)*
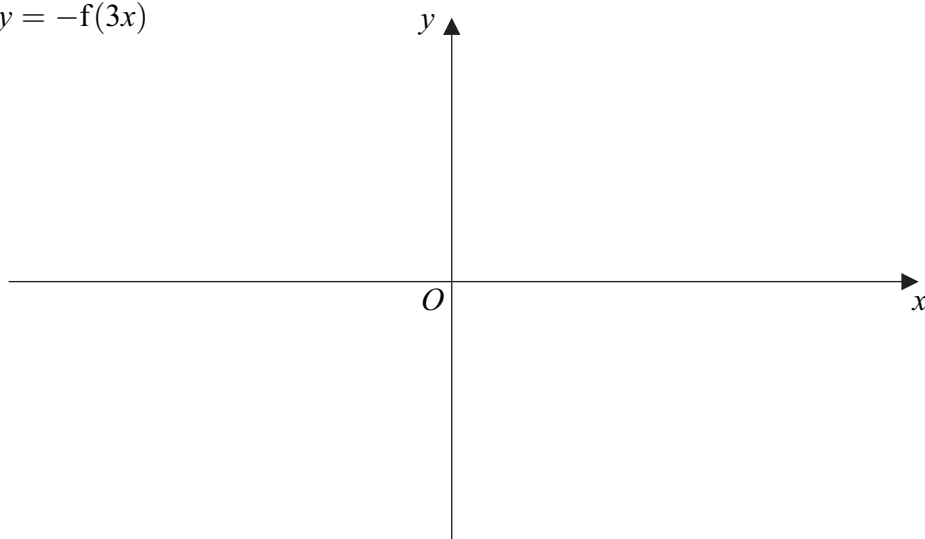
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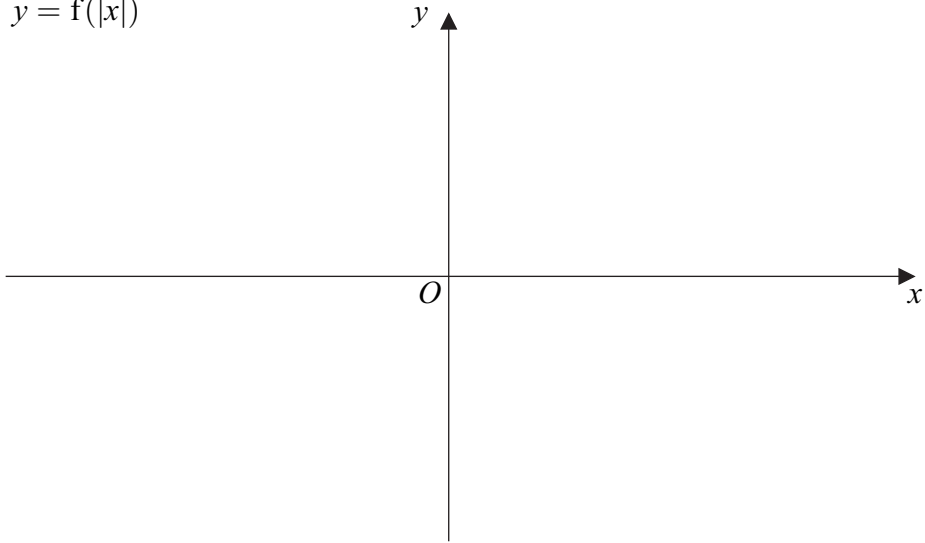REFERENCE

Answer space for question 7

(a)

Figure 1

$y = -f(3x)$

QUESTION PART REFERENCE	Answer space for question 7
(b)	**Figure 2**

$y = \text{f}(|x|)$

(c)

8 The curve with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.
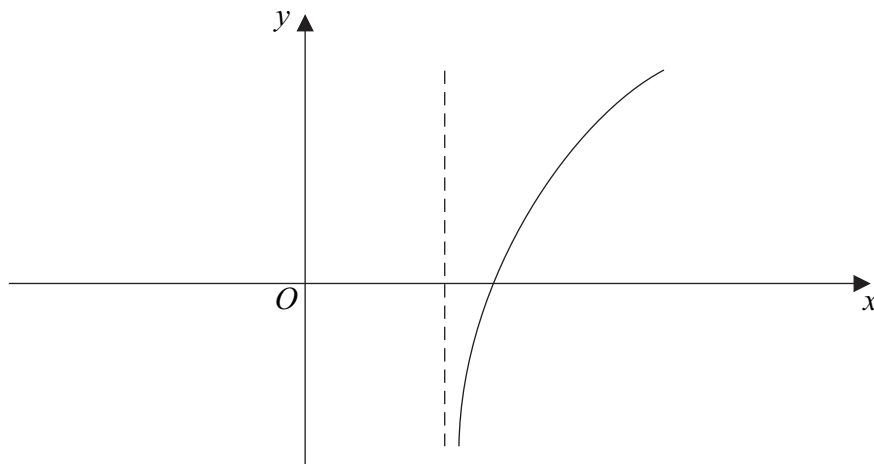
(a) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the range of f^{-1}. *(1 mark)*

(iii) Sketch, on the axes given on the opposite page, the curve with equation $y = \mathrm{f}^{-1}(x)$, indicating the value of the *y*-coordinate of the point where the curve intersects the *y*-axis. *(2 marks)*

(b) The function g is defined by

$$\mathrm{g}(x) = \mathrm{e}^{2x} - 4, \quad \text{for all real values of } x$$

(i) Find $\mathrm{gf}(x)$, giving your answer in the form $(ax - b)^2 - c$, where a, b and c are integers. *(3 marks)*

(ii) Write down an expression for $\mathrm{fg}(x)$, and hence find the exact solution of the equation $\mathrm{fg}(x) = \ln 5$. *(3 marks)*
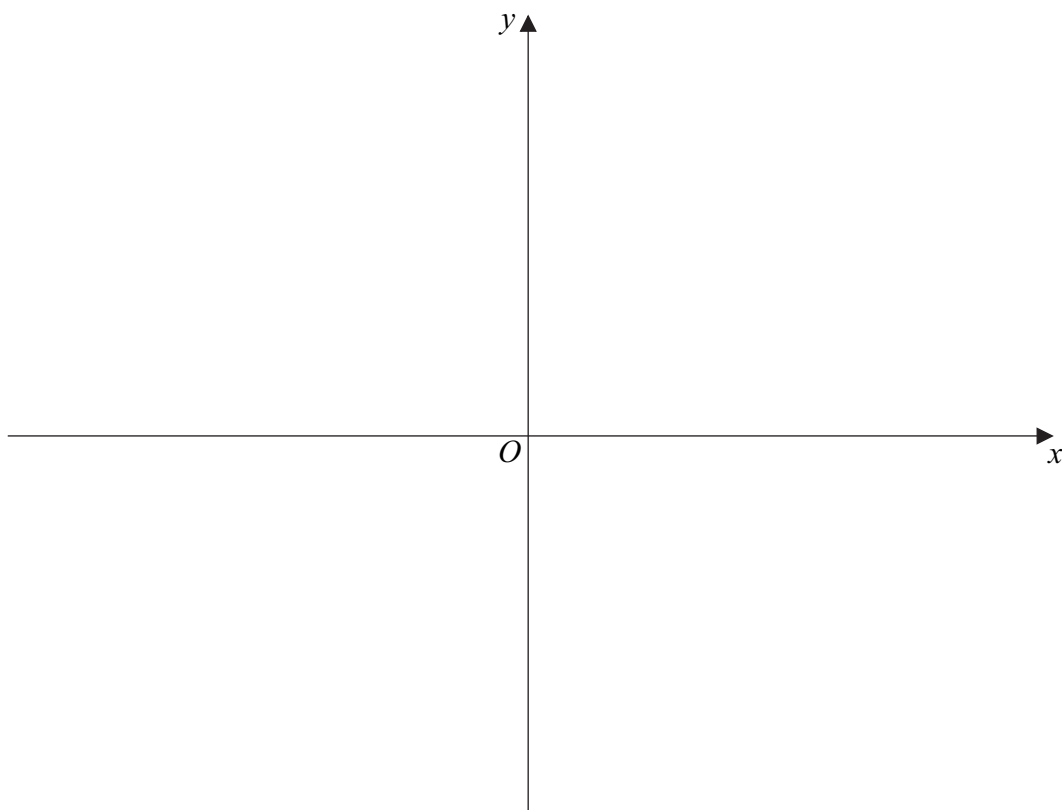
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(a)(iii)

y

O

x

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QUESTION PART REFERENCE	Answer space for question 8

QUESTION PART REFERENCE	Answer space for question 8

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9 The shape of a vase can be modelled by rotating the curve with equation $16x^2 - (y-8)^2 = 32$ between $y = 0$ and $y = 16$ completely **about the y-axis**.
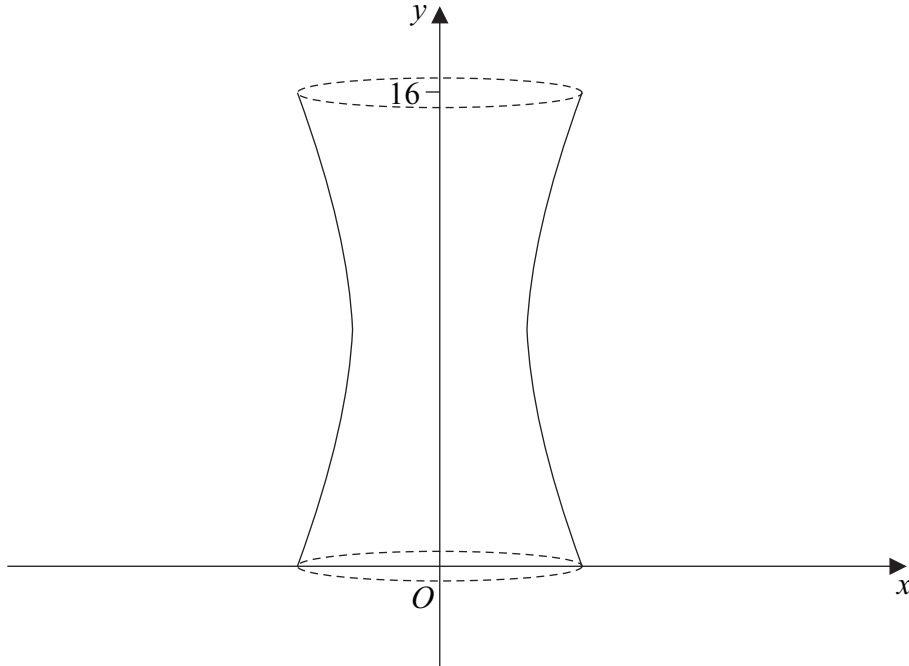
The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value.

(5 marks)

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10 (a) (i) By writing $\ln x$ as $(\ln x) \times 1$, use integration by parts to find $\int \ln x \, dx$. *(4 marks)*

(ii) Find $\int (\ln x)^2 \, dx$. *(4 marks)*

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$$

(7 marks)

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END OF QUESTIONS