



**General Certificate of Education (A-level)
June 2011**

Mathematics

MPC2

(Specification 6360)

Pure Core 2

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Copyright © 2011 AQA and its licensors. All rights reserved.

Copyright

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

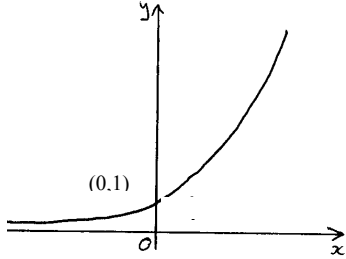
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\frac{10}{\sin \theta} = \frac{9}{\sin 54}$ $\sin \theta = \frac{10 \times \sin 54}{9} \left\{ = \frac{8.09...}{9} \right\} \left\{ = \frac{10}{11.12...} \right\}$ $\sin \theta = 0.898(9...) , \theta = 64.01(48...)$ $\theta = 64^\circ \text{ {to nearest degree}}$	M1	3	Sine rule, with $\sin \theta$ being the only unknown Correct rearrangement to ' $\sin \theta = \dots$ ' or to ' $\theta = \sin^{-1}(\dots)$ ' AG m1 must have been awarded and must see at least 3sf value either for $\sin \theta$ so that $0.898 \leq \sin \theta \leq 0.8993$ or for θ so that $64.0 \leq \theta \leq 64.1$ as well as seeing ' $\theta(\text{OE}) = 64$ '
		m1		
(b)	Angle $C = 180 - (54 + \theta) = 62$ {to 2sf} Area = $\frac{1}{2} \times 10 \times 9 \sin 62$ $= 39.73.. = 40$ {cm ² to nearest sq cm}	B1	3	$C = 62$. AWR T 62. PI if ' $C = 180 - (54 + \theta)$ ' and accurate later work. OE Ft c's value for C ($C \neq 54$, $C \neq \theta$) If not 40 condone a value 39.7 to 39.8 inclusive.
		M1		
Total			6	

Q	Solution	Marks	Total	Comments
2(a)	{Area of sector =} $\frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times 0.5$ $= 9$ (cm ²)	M1	2	$\frac{1}{2} r^2 \theta$ seen within (a) or used for the area Condone missing/incorrect units
		A1		
(b)(i)	{Arc =} $r\theta = 6 \times 0.5$ $= 3$ (cm)	M1	2	$r\theta$ seen within (b) or used for the arc length Condone missing/incorrect units
		A1		
(ii)	Perimeter of sector = $6 + 6 + \text{arc length}$ $= 15$ (cm) (= 5×3) Perimeter (of sector) = $5 \times (\text{length of arc})$	M1	2	PI by value of $12 + c$'s (b)(i) answer Completion, including concluding statement
		A1		
Total			6	

Q	Solution	Marks	Total	Comments
3(a)	$(2+x^2)^3$ $= [(2)^3] + 3(2)^2(x^2) + 3(2)(x^2)^2 + (x^2)^3$	M1	3	For either (1),3,3,(1) OE unsimplified or $\binom{3}{1}2^2x^2 + \binom{3}{2}2(x^2)^2$ OE. PI
	$p = 3(2)^2 = 12$	A1		AG Be convinced. Condone left as $12x^2$
	$q = 6$	B1		Accept left as $6x^4$
(b)(i)	$\int \frac{(2+x^2)^3}{x^4} dx = \int x^{-4}(8+12x^2+qx^4+x^6) dx$ or $\int \left(\frac{8}{x^4} + \frac{12}{x^2} + q + x^2 \right) dx$	M1	5	Uses (a) and either an indication that $\frac{1}{x^n} = x^{-n}$ in a product PI or cancelling to get at least 3 correct ft terms
	$\int (8x^{-4} + 12x^{-2} + q + x^2) dx$	A1F		Ft on c's non-zero q. PI by next line in solution
	$= \frac{8x^{-3}}{-3} + \frac{12x^{-1}}{-1} + qx + \frac{x^3}{3} \{+c\}$	M1		Correct integration of either $8x^{-4}$ or $12x^{-2}$; accept unsimplified
	$= \dots\dots\dots + 6x + \frac{x^3}{3} + c$	A1		Correct integration of both $8x^{-4}$ and $12x^{-2}$; accept unsimplified coefficients
	$(= -\frac{8}{3}x^{-3} - 12x^{-1} + 6x + \frac{x^3}{3} + c)$	B1F		For "6" $x + \frac{x^3}{3} + c$ simplified. The only ft is "6" replaced by c's value for q where q is a non-zero integer.
(b)(ii)	$\int_1^2 \frac{(2+x^2)^3}{x^4} dx = \left\{ -\frac{8}{3}(2)^{-3} - 12(2)^{-1} + 6(2) + \frac{2^3}{3} \right\} -$ $\left\{ -\frac{8}{3}(1)^{-3} - 12(1)^{-1} + 6(1) + \frac{1^3}{3} \right\}$	M1	2	Dealing correctly with limits; F(2)-F(1) (must have attempted integration to get F ie c's F is not just the integrand)
	$= \left(-\frac{1}{3} - 6 + 12 + \frac{8}{3} \right) - \left(-\frac{8}{3} - 12 + 6 + \frac{1}{3} \right)$	A1		OE exact answer eg 50/3. NMS scores 0
	$= 16\frac{2}{3}$			
Total			10	

Q	Solution	Marks	Total	Comments
4(a)		B1	2	Any graph only crossing the y-axis at (0, 1) stated /indicated ... (accept 1 on y-axis as equivalent) and not drawn below x-axis
		B1		Correct shaped graph, must clearly go below the intersection pt and an indication of correct behaviour of curve for large positive and large negative values of x. Ignore any scaling on axes.
(b)	Translation;	B1	2	Accept 'transl...' as equivalent [T or Tr is NOT sufficient]
	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$	B1		If vector not given, accept full equivalent to vector in words provided linked to 'transl./ move/shift' (B0B0 if >1 transformation)
(c)(i)	$4^x = (2^2)^x = 2^{2x} = (2^x)^2 = Y^2$ $2^{x+2} = 2^x \times 2^2 = 4Y$	M1	2	Justifying either $4^x = Y^2$ or $2^{x+2} = 4Y$ with no errors seen
	$4^x - 2^{x+2} - 5 = 0 \Rightarrow Y^2 - 4Y - 5 = 0$	A1		AG Be convinced; must have justified both of the above.
(ii)	$(Y - 5)(Y + 1) = 0$	M1	4	Correct factorising or use of quadratic formula or completing sq. PI by both solns 5 & -1 seen
	(Since) $2^x > 0$ (for all real x,) $2^x = 5$ so only one (real) solution	E1		Rejection of 2^x (condone Y) negative, with justification, (condone " 2^x not negative") followed by statement
	$\log 2^x = \log 5 \Rightarrow x \log 2 = \log 5$	M1		Eqn of form $p^x = q \Rightarrow x \log p = \log q$ provided $p > 0$ & $q > 0$ OE eg $x = \log_2 5$
	$x = 2.3219... = 2.322 \text{ (to 3dp)}$	A1		Condone > 3dp but must see explicit use of logs and must only be the one solution.
	Total		10	

Q	Solution	Marks	Total	Comments
5(a)	$\frac{dy}{dx} = 6 - 3x^{\frac{1}{2}}$	B1	3	For either 6 or $6x^0$
		M1		$Ax^{\frac{3}{2}-1}$, $A \neq 0$ OE
		A1		$6 - 3x^{\frac{1}{2}}$ or $6 - 3\sqrt{x}$ with no '+c' [If unsimplified here, A1 can be awarded retrospectively if correct simplified expression is seen explicitly in (b)(i).]
(b)(i)	$6 - 3x^{\frac{1}{2}} = 0$ $x^{\frac{1}{2}} = 2 \Rightarrow x = 2^2$ $M(4, 8)$	M1	3	Equating c's $\frac{dy}{dx}$ to 0 PI by correct ft rearrangement of c's $dy/dx=0$
		m1		$x^{\frac{1}{2}} = k$ ($k > 0$), to $x = k^2$. PI by correct value of x if no error seen
		A1		SC If M0 award B1 for (4, 8)
(ii)	Eqn of normal at M is $x = 4$	B1F	1	Ft on $x = c$'s x_M
(c)(i)	When $x = \frac{9}{4}$, $\frac{dy}{dx} = 6 - 3 \times \frac{3}{2} = \frac{3}{2}$ Gradient of normal at $P = -\frac{2}{3}$ Eqn of normal: $y - \frac{27}{4} = -\frac{2}{3}\left(x - \frac{9}{4}\right)$ $12y - 81 = -8x + 18 \Rightarrow 8x + 12y = 99$	M1	4	Attempt to find $\frac{dy}{dx}$ when $x = \frac{9}{4}$
		m1		$m \times m' = -1$ used
		A1		ACF eg $y = -\frac{2}{3}x + \frac{33}{4}$
		A1		Coeffs and constant must now be positive integers, but accept different order eg $12y + 8x = 99$
(ii)	$8(4) + 12y = 99$ $R\left(4, \frac{67}{12}\right)$	M1	2	Solving c's answer (b)(ii), (must be in form $x = \text{positive const}$), with c's answer (c)(i). PI by correct earlier work and <u>correct</u> coordinates for R .
		A1		Accept 5.58 or better as equivalent to $\frac{67}{12}$
Total			13	

Q	Solution	Marks	Total	Comments
6(a)	$h = 0.5$	B1		$h = 0.5$ stated or used. (PI by x -values 0, 0.5, 1, 1.5, 2 provided no contradiction)
	$f(x) = \sin x$ $I \approx h/2 \{ \dots \}$ $\{ \dots \} = f(0) + f(2) + 2[f(0.5) + f(1) + f(1.5)]$	M1		OE summing of areas of 'trapezia'..
	$\{ \dots \} =$ $0 + 0.90929.. + 2[0.4794.. + 0.84147.. + 0.99749..]$	A1		Min. of 2dp values rounded or truncated. Can be implied by later correct work provided >1 term or a single term which rounds to 1.39
	$\{ \dots \} = 0.90929.. + 2[2.318..] = \{0.90929.. + 4.636..\}$ $(I \approx) 0.25[5.546..] = 1.3865.. = 1.39$ (to 3sf)	A1	4	CAO Must be 1.39
(b)	Stretch(I) in y -direction(II) scale factor 2(III)	M1	2	Need (I) and either (II) or (III) All correct. Need (I) and (II) and (III) [>1 transformation scores 0/2]
		A1		
(c)	$\frac{\sin x}{\cos x} = \frac{1}{2}; \tan x = 0.5$ $\tan x = 0.5$ $x = \alpha$ or $\pi + \alpha$ where $\alpha = \tan^{-1}(k)$ $x = 0.464, 3.61$	M1		$\frac{\sin x}{\cos x} = \tan x$ used to get $\tan x = k$ or identity $\cos^2 x + \sin^2 x = 1$ used to get either $\sin^2 x = p$ or to get $\cos^2 x = q$, (p and q must be between 0 and 1)
		A1		Either $\tan x = \frac{1}{2}$ or $\cos x = \pm \sqrt{\frac{4}{5}}$ ($= \pm 0.894..$) or $\sin x = \pm \sqrt{\frac{1}{5}}$ ($= \pm 0.447..$)
		m1		Correct method to find 2 nd angle. Any in wrong ft quadrants then m0. In case of squaring method candidates must also have rejected the extra 'quadrants' for the m1. Condone degrees or mixture
		A1	4	Both. Condone $>3sf$ [0.463(6..), 3.60(5.. or 6..)] Accept pair of truncated values [0.463, 3.60] Ignore any answers outside interval 0 to 6.28
Total			10	

Q	Solution	Marks	Total	Comments
7(a)	$48 = 60p + q$ $12 = 12p + q$ $36 = 48p$ or $p = \frac{36}{48}$ $p = \frac{3}{4}$ $q = 3$	M1 M1 m1 A1 B1	5	M1 for each equation in ACF (Condone embedded values for the M1M1) Valid method to solve the correct two simultaneous eqns in p and q to at least the stage $ap = b$ or $cq = d$ AG (condone if left as equiv. decimal) Can award if seen explicitly in (b) and no contradiction [ie not attempted in (a)]
(b)	$u_3 = 36 + q = 39$	B1F	1	If not 39, ft on $(36 + c's q)$
Total			6	

Q	Solution	Marks	Total	Comments
8	$\dots = 9\sin^2 x + 6\sin x \cos x + \cos^2 x +$ $\sin^2 x - 6\sin x \cos x + 9\cos^2 x$ $\dots = 10\cos^2 x + 10\sin^2 x$ $= 10(1 - \sin^2 x) + 10\sin^2 x$ $= 10$ (which is an integer)	M1 A1 M1 A1	4	Attempt at expanding both sets of brackets. Minimum requirement either one of the two expansions correct or 4 of these 6 terms seen. Expanding and simplifying the given expression in one step to get the correct two terms scores this M1 and next A1 Either correct pair of expansions and simplification to remove $\sin x \cos x$ terms or full collecting of like terms within the original correct expansion $\cos^2 x + \sin^2 x = 1$ clearly used. If identity is applied correctly, but not directly, it must be stated at the relevant point in the proof. CSO [all previous 3 marks must have been scored] Condone absence of statement after 10 obtained correctly
Total			4	

Q	Solution	Marks	Total	Comments
9(a)	$\{S_\infty =\} \frac{a}{1-r} = \frac{12}{1-\frac{3}{8}}$	M1		$\frac{a}{1-r}$ <u>used</u>
	$\{S_\infty =\} 19.2$	A1	2	19.2 OE NMS mark as 2/2 or 0/2
(b)	$\{6\text{th term} =\} ar^{6-1}$	M1		Stated or used
	$= 12 \times \left(\frac{3}{8}\right)^5 = 2 \times 2 \times 3 \times \left(\frac{3}{2 \times 2 \times 2}\right)^5$	m1		Changing 8 and 12 in correct expression to correct products/powers of 2 and 3
	$= \frac{2 \times 2 \times 3 \times 3^5}{(2^3)^5} = \frac{2^2 \times 3^6}{2^{15}} = \frac{3^6}{2^{13}}$	A1	3	AG Be convinced
(c)(i)	$\{u_n =\} 12 \times \left(\frac{3}{8}\right)^{n-1}$	B1	1	OE. eg $32(3/8)^n$
(ii)	$\log u_n = \log 12 + \log \left(\frac{3}{8}\right)^{n-1}$			<u>Log laws</u> $\log(PQ) = \log P + \log Q$; $\log\left(\frac{P}{Q}\right) = \log P - \log Q$ $\log(P)^k = k \log P$
	$\log u_n = \log 12 + (n-1) \log \left(\frac{3}{8}\right)$			
	$\log u_n = \log 12 + (n-1)[\log 3 - \log 8]$	M1		Using (c)(i) and taking logs: one log law used correctly, on a correct expression for u_n .
		M1		a second different log law used correctly, indep of prev M error and ft on cand's (c)(i) provided cand's u_n expression has a power involving n .
	$\log u_n = \log 3 + 2 \log 2 + (n-1)[\log 3 - 3 \log 2]$	m1		A third different log law used correctly (or equivalent valid step) to reach a correct RHS whose terms are all multiples of $\log 2$ and $\log 3$. Dep on both prev two Ms
	$\log u_n = n \log 3 - 3n \log 2 + 5 \log 2$ $\log_a u_n = n \log_a 3 - (3n-5) \log_a 2$	A1	4	CSO AG Be convinced, no slips although we will condone the absence of the bases a even in the final line.
	Total		10	