

General Certificate of Education
June 2007
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. (2 marks)

(ii) Hence find an equation of the line AB , giving your answer in the form $ax + by = c$, where a , b and c are integers. (2 marks)

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB . (2 marks)

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k . (2 marks)

2 (a) Express $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$ in the form $n\sqrt{7}$, where n is an integer. (3 marks)

(b) Express $\frac{\sqrt{7} + 1}{\sqrt{7} - 2}$ in the form $p\sqrt{7} + q$, where p and q are integers. (4 marks)

3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. (2 marks)

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. (2 marks)

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. (1 mark)

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. (3 marks)

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. (4 marks)

- 4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \quad 0 \leq t \leq 4$$

- (a) Find:

(i) $\frac{dy}{dt}$; (3 marks)

(ii) $\frac{d^2y}{dt^2}$. (2 marks)

- (b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. (4 marks)
- (c) Find the rate of change of y with respect to t when $t = 1$. (2 marks)
- (d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. (2 marks)

- 5 A circle with centre C has equation $(x + 3)^2 + (y - 2)^2 = 25$.

- (a) Write down:

(i) the coordinates of C ; (2 marks)

(ii) the radius of the circle. (1 mark)

- (b) (i) Verify that the point $N(0, -2)$ lies on the circle. (1 mark)

(ii) Sketch the circle. (2 marks)

(iii) Find an equation of the normal to the circle at the point N . (3 marks)

- (c) The point P has coordinates $(2, 6)$.

(i) Find the distance PC , leaving your answer in surd form. (2 marks)

(ii) Find the length of a tangent drawn from P to the circle. (3 marks)

Turn over for the next question

Turn over ►

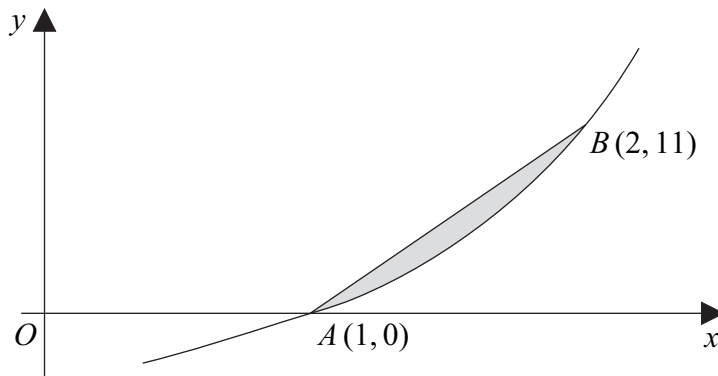
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. (2 marks)

(ii) Express $f(x)$ in the form $(x - 1)(x^2 + px + q)$, where p and q are integers. (2 marks)

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. (3 marks)

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x -axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5) dx$. (3 marks)

(ii) Hence find the area of the shaded region bounded by the curve and the line AB . (4 marks)

7 The quadratic equation

$$(2k - 3)x^2 + 2x + (k - 1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leq 0$. (3 marks)

(b) (i) Factorise $2k^2 - 5k + 2$. (1 mark)

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leq 0 \quad (3 \text{ marks})$$

END OF QUESTIONS