AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

A-LEVEL
MATHEMATICS

Unit Mechanics 2B

Monday 26 June 2017 Afternoon Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 A ball of mass $3\ \mathrm{kg}$ is thrown from a hot-air balloon. When the ball is thrown, it is 50 metres above the ground. The ground is horizontal. The ball initially moves with speed $12\ \mathrm{m\ s^{-1}}$.

In this question, model the ball as a particle and ignore air resistance.

(a) Find the initial kinetic energy of the ball.

[2 marks]

(b) Find the kinetic energy of the ball when it hits the ground.

[3 marks]

(c) Hence find the speed of the ball when it hits the ground.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	**Answer space for question 1**

Turn over ►

2 A particle moves on a horizontal plane in a straight line. At time t seconds, it has velocity $v \text{ m s}^{-1}$ where $v = 4t^3 - 3\sin 4t + 8, \ t \geqslant 0$.

(a) (i) Find an expression for the acceleration of the particle at time t.

[2 marks]

(ii) Find the acceleration of the particle when $t = \dfrac{\pi}{4}$.

[2 marks]

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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QUESTION PART REFERENCE	Answer space for question 2

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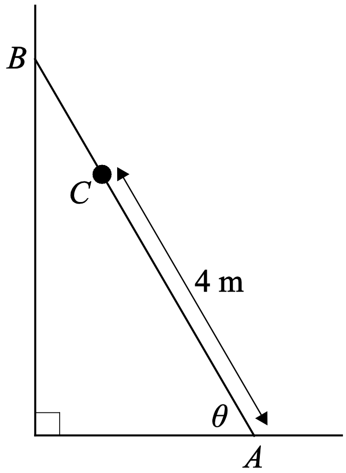
3 A uniform ladder AB of length 7 metres and mass $15\ \mathrm{kg}$ rests with its foot A on a rough horizontal floor. Its top B is leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the floor is θ.

A man of mass $70\ \mathrm{kg}$ is standing at point C on the ladder so that the distance AC is 4 metres. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the horizontal floor is 0.3 . Model the man as a particle at C.



Find the angle θ.

[6 marks]

QUESTION PART REFERENCE	Answer space for question 3

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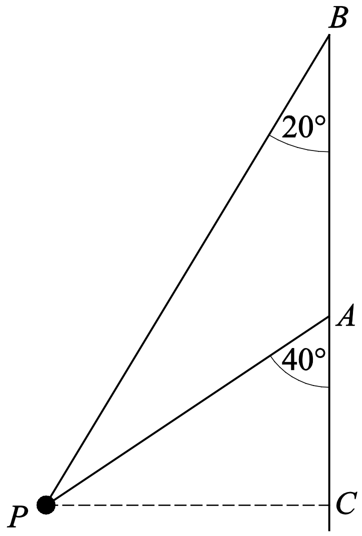
4 Two light inextensible strings each have one end attached to a particle P of mass 6 kg. The other ends of the strings are attached to the fixed points A and B. The point B is vertically above the point A. The particle moves at a constant speed in a horizontal circle. The centre, C, of this circle is directly below the point A. The two strings are inclined at $20°$ and $40°$ to the vertical, as shown in the diagram. Both strings are taut. As the particle moves in the horizontal circle, the tension in the string BP is 30 N.



(a) Find the tension in the string AP.

[3 marks]

(b) The speed of the particle is 8 m s^{-1}.

Find the length of CP, the radius of the horizontal circle.

[4 marks]

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5 A car, of mass 1600 kg, has a maximum speed of $45\ \mathrm{m\ s^{-1}}$ on a straight horizontal road.

When the car travels at speed $v\ \mathrm{m\ s^{-1}}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the car is $81\,000$ watts.

[2 marks]

(b) The car is travelling along this straight horizontal road at a speed of $30\ \mathrm{m\ s^{-1}}$.

Find the maximum possible acceleration of the car when it is travelling at this speed.

[2 marks]

(c) The car starts to descend a hill which is inclined at an angle of θ to the horizontal.

If the maximum possible constant speed of the car as it travels in a straight line down the hill is $55\ \mathrm{m\ s^{-1}}$, find the angle θ.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 5

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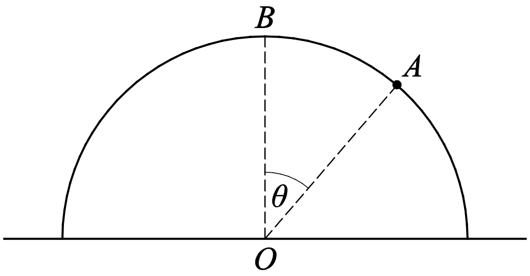
6 A particle is placed at the point B on the top of a smooth upturned hemisphere of radius 4 metres and centre O. The particle is set into motion with an initial horizontal velocity of $U \text{ m s}^{-1}$. When the particle is at point A on the surface of the hemisphere, the angle between OA and OB is θ and the particle has speed $v \text{ m s}^{-1}$.



If the particle leaves the surface of the hemisphere when $\theta = 35°$, find the value of U.

[7 marks]

QUESTION PART REFERENCE	Answer space for question 6

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7 Initially a speedboat is travelling at a speed of 15 m s^{-1} in a straight line across a lake. At time t seconds later, the speedboat has velocity $v \text{ m s}^{-1}$ and the engine is producing a constant force of 600 newtons.

The speedboat experiences a resistance force of magnitude $90v$ newtons.

The mass of the speedboat plus passengers is 450 kg.

Assume that the water in the lake is still.

(a) Show that $\dfrac{dv}{dt} = -\dfrac{3v - 20}{15}$.

[1 mark]

(b) Find an expression for v in terms of t.

[6 marks]

(c) Find the time taken for the speed of the speedboat to reduce to 10 m s^{-1}.

[2 marks]

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8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\frac{\lambda e}{l}$ when its extension is $e \geqslant 0$.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\frac{\lambda e^2}{2l}$.

[3 marks]

(b) A particle of mass $10\ \mathrm{kg}$ is attached to one end of a light elastic string of natural length 0.8 metres and modulus of elasticity $250\ \mathrm{N}$. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O.

[2 marks]

(ii) The particle is pulled down and held at the point P, which is 1.4 metres vertically below O.

Find the elastic potential energy of the string when the particle is in this position.

[2 marks]

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\ \mathrm{m\ s^{-1}}$ when it is x metres **above *P***.

Show that, while the string is taut, $20v^2 = 358x - 625x^2$.

[5 marks]

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut.

[2 marks]

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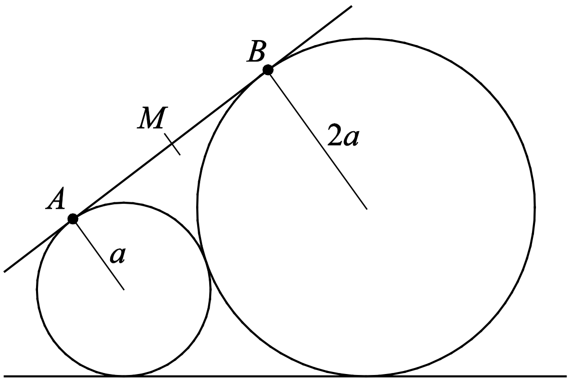
9 Two spheres, one of radius a and the other of radius $2a$, are fixed in space with their bases on a horizontal plane so that the two spheres are in contact with each other.

A rod is positioned so that the rod is a tangent to the sphere of radius a at the point A and to the sphere of radius $2a$ at the point B.

The coefficient of friction between the rod and the sphere of radius $2a$ at B is μ, and the coefficient of friction between the rod and the sphere of radius a at A is 2μ.

The length of the rod is $4a$, with its centre of mass M being equidistant from A and B.



(a) The rod is inclined at an angle of 2θ to the horizontal.

Prove that $\sin 2\theta = \dfrac{4\sqrt{2}}{9}$.

[2 marks]

(b) Find μ if the rod is in limiting equilibrium at both A and B.

[6 marks]

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END OF QUESTIONS

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