AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 5

Friday 15 June 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 A simple pendulum has a string of length l. A student wants to increase the period of the pendulum by 5%.

Find the percentage change in the length of the string that is needed to achieve the 5% increase.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

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2 A particle, of mass $1.5\,\text{kg}$, is attached to one end of a light spring. The other end of the spring is attached to a fixed point, O. When in equilibrium, the particle is below O. The particle moves with simple harmonic motion in a vertical line through O. The maximum speed of the particle is $0.16\,\text{m s}^{-1}$. When the particle is 0.02 metres from its equilibrium position, the speed of the particle is $\frac{\sqrt{15}}{25}\,\text{m s}^{-1}$

(a) Find the period of the motion.

[5 marks]

(b) Show that the stiffness of the spring is $6\,\text{N m}^{-1}$.

[4 marks]

(c) Find the maximum tension in the spring during the motion.

[5 marks]

QUESTION PART REFERENCE

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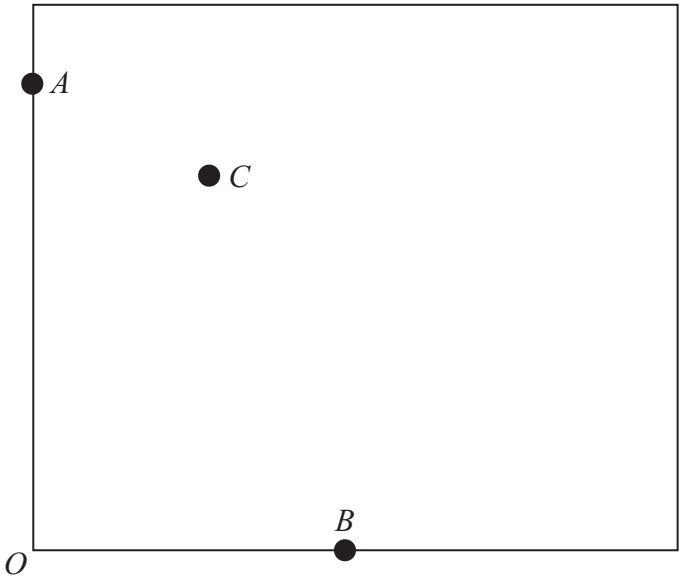
3 Three particles, A, B and C, are fixed to a light lamina at the points listed below:

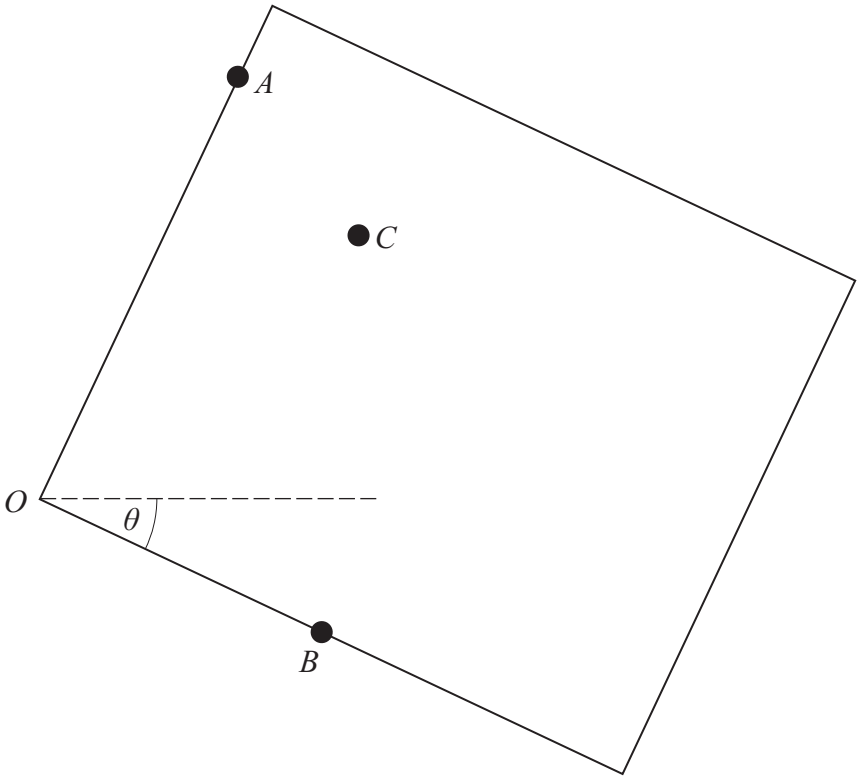
Particle	Mass	Distance from OA	Distance from OB
A	$2m$	0	4
B	$3m$	3	0
C	$5m$	2	$2\sqrt{3}$

The lamina is smoothly pivoted at the corner O. The lamina is shown in the diagram below.



The lamina remains in the vertical plane, that contains the points A, B, C and O.

The side OB is an angle θ radians below the horizontal, as shown in the diagram below.



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The potential energy is taken to be zero at the level of the point O.

(a) Show that the total potential energy of the system, V, is given by

$$V = mg((8 + 10\sqrt{3})\cos\theta - 19\sin\theta)$$

[5 marks]

(b) Using V, find the values of θ for which the lamina is in equilibrium.

[4 marks]

(c) Determine the nature of each equilibrium position. Justify your answer.

[4 marks]

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4 A light spring has natural length 0.5 metres and modulus of elasticity $20\,\mathrm{N}$. One end of the spring is attached to a peg. A sphere of mass $1.6\,\mathrm{kg}$ is attached to the other end of the spring. Model the sphere as a particle and assume that no resistance forces act on the sphere.

(a) Assume that the peg remains at rest. Find the length of the spring when the sphere is in equilibrium, directly below the peg.

[3 marks]

(b) The peg then starts to move up and down. It moves so that its displacement below its initial position, O, is given by $0.1\sin(10t)$.

The displacement of the sphere below O at time t seconds is x metres.

(i) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 25x = 22.3 + 2.5\sin(10t)$$

[4 marks]

(ii) Find x in terms of t.

[12 marks]

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5 A particle moves on the curve defined by $r = 1 + \sin\theta$ and $\theta = \frac{2t}{5}$

(a) Show that the speed of the particle is proportional to $\sqrt{r}$

[5 marks]

(b) Find the maximum and minimum magnitudes of the acceleration of the particle.

[7 marks]

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6 A spherical hailstone falls vertically through a stationary cloud. As it falls the radius of the hailstone increases at a rate proportional to the radius, r metres. When the radius of the hailstone is 0.001 metres the radius is increasing at $0.002\,\mathrm{m\,s^{-1}}$. At time $t = 0$, the hailstone has velocity $U\,\mathrm{m\,s^{-1}}$.

Assume that no resistance forces act on the hailstone.

Assume that the density, $\rho\,\mathrm{kg\,m^{-3}}$, of the hailstone is constant.

(a) At time t seconds the velocity of the hailstone is $v\,\mathrm{m\,s^{-1}}$. Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - 6v$$

[7 marks]

(b) Find v in terms of g, U and t.

[5 marks]

(c) What happens to the speed of the hailstone as t becomes large?

[1 mark]

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