

**MATHEMATICS**  
**Unit Mechanics 4**

**MM04**

Friday 19 June 2009 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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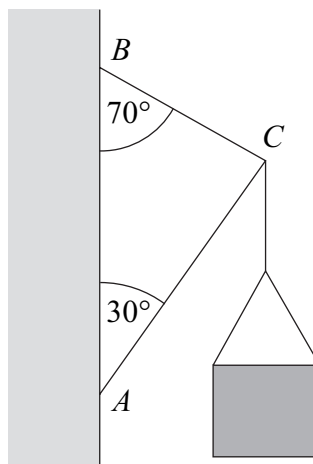
1 The cylindrical drum in a spin dryer rotates about its vertical axis. Initially, the drum is at rest. It then rotates with a constant acceleration and reaches its maximum angular speed of 1200 revolutions per minute in 10 seconds.

(a) Show that the magnitude of the angular acceleration is  $4\pi \text{ rad s}^{-2}$ . (4 marks)

(b) A couple of constant magnitude  $100\pi \text{ N m}$  causes the drum to rotate with this angular acceleration. Find the moment of inertia of the drum about the axis of rotation. (2 marks)

2 Two light smoothly-jointed rods,  $AC$  and  $BC$ , support a shop sign of mass 20 kg.

The two rods are smoothly hinged to a vertical wall at  $A$  and  $B$ , with  $B$  directly above  $A$ . Angle  $BAC$  is  $30^\circ$  and angle  $ABC$  is  $70^\circ$ . The shop sign hangs in equilibrium from  $C$ , as shown in the diagram.



Find the magnitudes of the forces in rods  $AC$  and  $BC$ , stating whether the rods are in tension or compression. (7 marks)

3 The forces  $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ ,  $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$  and  $4\mathbf{j} - 2\mathbf{k}$  act at the points with coordinates  $(2, 1, 0)$ ,  $(1, 13, -2)$  and  $(6, 4, -7)$  respectively. The resultant of the three forces is a single force  $\mathbf{F}$ .

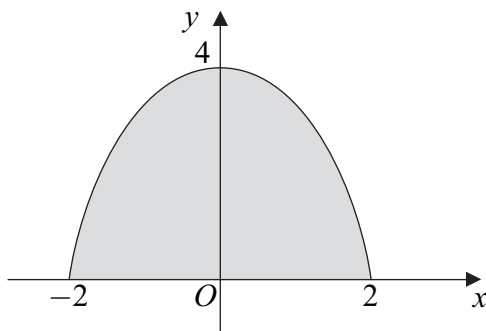
(a) Show that the magnitude of  $\mathbf{F}$  is 7. (3 marks)

(b) The point  $P$  has coordinates  $(3, 6, -3)$ .

(i) Find the moment of the force  $4\mathbf{j} - 2\mathbf{k}$  about  $P$ . (4 marks)

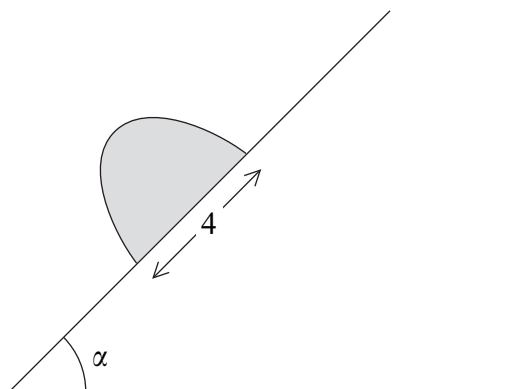
(ii) Given that the resultant of the two forces  $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$  and  $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$  acts through  $P$ , state the moment of  $\mathbf{F}$  about  $P$ , giving a reason for your answer. (2 marks)

4 (a) A uniform lamina is bounded by the curve  $y = 4 - x^2$  and the  $x$ -axis, as shown in the diagram.



Given that the area of the lamina is  $\frac{32}{3}$  square units, find the  $y$ -coordinate of the centre of mass of the lamina. (5 marks)

(b) The cross-section of a uniform prism is the same shape as the lamina in part (a). The prism is placed on a plane inclined at an angle  $\alpha$  to the horizontal with the rectangular base of the prism in contact with the inclined plane, as shown in the diagram.

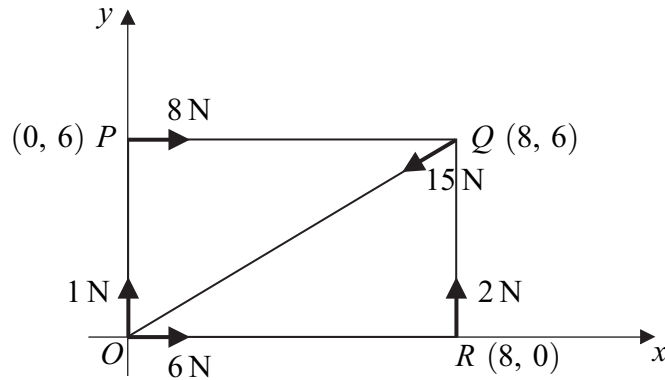


Given that the prism is just about to topple and that no slipping occurs, find the value of  $\alpha$ , giving your answer to the nearest degree. (4 marks)

Turn over ►

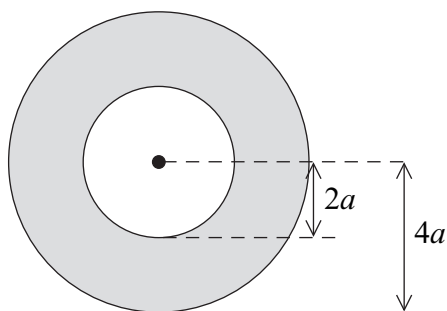
- 5 The points  $O$ ,  $P$ ,  $Q$  and  $R$  have coordinates  $(0, 0)$ ,  $(0, 6)$ ,  $(8, 6)$  and  $(8, 0)$  respectively. The units of length are metres.

A force of 1 N acts at  $O$  along  $OP$ .  
 A force of 2 N acts at  $R$  along  $RQ$ .  
 A force of 6 N acts at  $O$  along  $OR$ .  
 A force of 8 N acts at  $P$  along  $PQ$ .  
 A force of 15 N acts at  $Q$  along  $QO$ .



- (a) Show that the resultant of the five forces has magnitude  $2\sqrt{10}$  N. (5 marks)
- (b) An anticlockwise couple of magnitude 20 N m together with these five forces is equivalent to a single force of magnitude  $2\sqrt{10}$  which has a line of action passing through the point  $(0, d)$ .
- (i) Find  $d$ . (5 marks)
- (ii) Determine the equation of the line of action of the resultant, giving your answer in vector form. (3 marks)

- 6 (a) Show, by integration, that the moment of inertia of a uniform disc, of mass  $m$  and radius  $r$ , about an axis through its centre and perpendicular to the plane of the disc is  $\frac{mr^2}{2}$ . (5 marks)
- (b) A disc, of radius  $2a$ , is removed from the centre of a uniform disc, of radius  $4a$ . The resulting ring has mass  $M$  and is shown in the diagram.



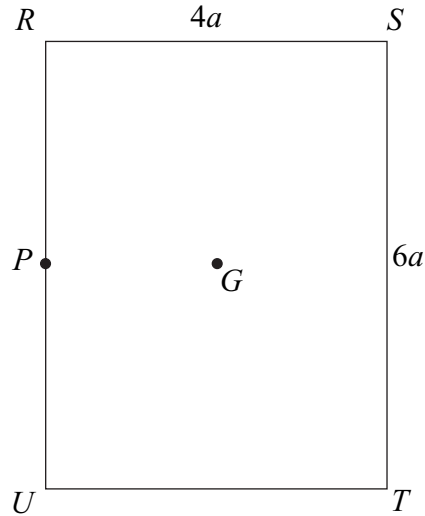
Using the result from part (a), or otherwise, show that the moment of inertia of the ring about an axis through its centre and perpendicular to the plane of the ring is  $10Ma^2$ . (5 marks)

- (c) Determine the moment of inertia of the ring about an axis along a diameter, stating any theorem that you use. (3 marks)

**Turn over for the next question**

**Turn over ►**

- 7 A uniform rectangular lamina,  $RSTU$ , has mass  $M$ , with  $RS = 4a$  and  $ST = 6a$ . The centre of mass of the lamina is  $G$ , and the mid-point of  $RU$  is  $P$ .



- (a) (i) Show that the moment of inertia of the rectangular lamina about an axis perpendicular to its plane and passing through  $G$  is  $\frac{13Ma^2}{3}$ . (2 marks)
- (ii) Hence find the moment of inertia of the lamina about an axis perpendicular to its plane and passing through  $P$ . (2 marks)
- (b) The lamina is smoothly hinged at  $P$ . It is free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passes through  $P$ . Initially, the lamina is held with  $PG$  horizontal and then released. At time  $t$  after release,  $PG$  makes an angle  $\theta$  with the horizontal.
- (i) Show that  $\dot{\theta}^2 = \frac{12g \sin \theta}{25a}$ . (5 marks)
- (ii) Hence, or otherwise, determine an expression for  $\ddot{\theta}$ , in terms of  $a$ ,  $g$  and  $\theta$ . (3 marks)
- (iii) Show that the magnitude of the component in the direction  $GP$  of the force at  $P$  which the hinge exerts on the lamina is  $\frac{49Mg \sin \theta}{25}$ . (3 marks)
- (iv) Find, in terms of  $M$ ,  $g$  and  $\theta$ , the magnitude of the component in the direction  $PR$  of the force at  $P$  which the hinge exerts on the lamina. (3 marks)

**END OF QUESTIONS**

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