Version 1.0: 0606

General Certificate of Education

Mathematics 6360

MM04 Mechanics 4

Mark Scheme

2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

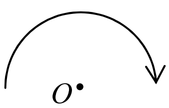
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

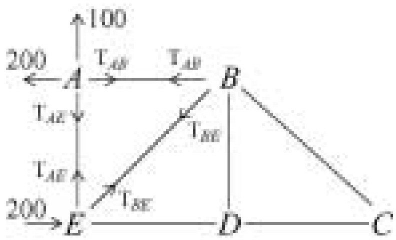
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

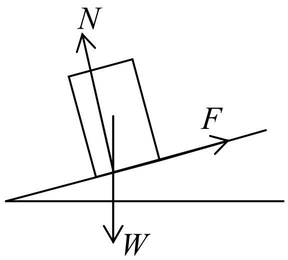
MM04

Q	Solution	Marks	Total	Comments
1(a)	**i** **j** **k** 2 3 -5 a b $+5$ -2 0 0 0	M1		$\sum$ forces $=0$
	$\Rightarrow b=-5,\ a=2$	A1	2	Both correct
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 1 \\ -5 & 5 & -2 \end{vmatrix}$	M1 A1		Intention to find $\mathbf{r}\times\mathbf{F}$ $[\text{or } \mathbf{F}\times\mathbf{r}]$ Two correct non zero determinants attempted
	$=-2\mathbf{k}-9\mathbf{i}-7\mathbf{j}+5\mathbf{k}$	A1		At least two non zero terms correct
	$=-9\mathbf{i}-7\mathbf{j}+3\mathbf{k}$	A1	4	Fully simplified. Lose 1 if $\mathbf{F}\times\mathbf{r}$ found
	Total		**6**	
2(a)(i)	$\begin{pmatrix} 8+6+-2+0 \\ 4+5-2-2 \end{pmatrix}=\begin{pmatrix} 12 \\ 5 \end{pmatrix}$	M1 A1		Adding forces $R_x=12$ $R_y=5$
	$\lvert \mathrm{F}\rvert=\sqrt{12^2+5^2}=13$	A1√	3	Follow through mis-totals – use of Pythagoras' rule
(ii)	Moment of forces about O $=-6(3)-2(3)-2(4)+2(4)$ $=-18-6-8+8$ $=-24$	M1 A1		One term correct All terms correct $[\text{NB} \pm \text{can be reversed}]$
	$-12d=-24$	M1		Equation formed $R_x d=$ Total moment
	$\therefore d=2$	A1	4	Printed answer
	Alternative Total Moment = –24	(M1) (A1)		
	Equation of line of action is $y=\frac{5}{12}\left(x+\frac{24}{5}\right)$	(M1)		
	When $x=0, y=2$	(A1)		
(b)	$\lvert c\rvert=24$	B1√		Follow through (a)(ii) ie $\lvert$their total moment$\rvert$
		B1√	2	Accept 'clockwise' Follow through their direction from (a)(ii)
	Total		**9**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Take moments at A for system			
	$P(l) = 100(2l)$	M1		Moments for system
	$\Rightarrow P = 200\text{ N}$	A1	2	
(b)(i)	For whole system to be in equilibrium;			
	$X = 200$ left $\quad Y = 100$ up	M1		Balances system and uses Pythagoras' rule
	$\therefore$ magnitude $= 100\sqrt{5}\text{ N}$	A1	2	Accept 224, $\sqrt{50000}$
(ii)	Angle $= \tan^{-1}\left(\frac{100}{200}\right) = 27^\circ$	B1	1	Condone unrounded answers
(c)				
	By considering forces at A			
	$T_{AB} = 200$	B1		
	$T_{AE} = 100$	B1		
	By considering forces at E			
	Vertically, $T_{AE} + T_{BE}\cos 45^\circ = 0$	M1		Form equation
	$\left\| T_{BE} \right\| = 100\sqrt{2}$	A1		Accept 141
	AB in tension, *AE* in tension, *BE* in compression	B1	5	All correct
	Total		**10**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)		B1 B1	 2	Three forces **Alternative**: N/F combined Through one point
(b)	About to topple $\Rightarrow \tan\alpha = \frac{d}{3d}$ $\therefore \alpha = \tan^{-1}\left(\frac{1}{3}\right)$ $= 18.4^\circ$	M1 A1 A1	 3	Use of tan α $\frac{d}{3d}$ used or $\frac{3d}{d}$ Accept 18°
(c)	Parallel to plane $W \sin\alpha = F$ Perpendicular to plane $W \cos\alpha = N$ Law of friction $F \le \mu N$ $\mu = \frac{2}{9}\ \therefore W \sin\alpha \le \frac{2}{9} W \cos\alpha$ $\tan\alpha \le \frac{2}{9}$ $\frac{2}{9} < \frac{1}{3}$ or $13^\circ < 18^\circ$ Slide first	M1 A1 M1 A1 A1	 5	Attempt to resolve parallel <u>or</u> perpendicular to the plane Both correct Law of friction used $\tan\alpha$ or α obtained or inequality stated Comparison and conclusion – correct answer only [N.B. Accept (b) and (c) in any order]
	Total		**10**	

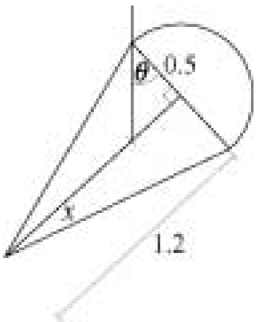
MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	For cylinder, $C = I\ddot{\theta}$			
	$\Rightarrow Tr = 4mr^2\ddot{\theta}$	M1A1		Use of $C = I\ddot{\theta}$
	$\therefore T = 4mr\ddot{\theta}$ **1**			
	For particle $2mg - T = 2mr\ddot{\theta}$ **2**	M1 A1A1		Attempt to use $F = ma$ A1 RHS A1 LHS
	1 + 2 $2mg = 6mr\ddot{\theta}$			
	$\ddot{\theta} = \dfrac{g}{3r}$	A1	6	Printed answer
	Alternative:			
	KE of cylinder $= \frac{1}{2}I\dot{\theta}^2$			
	$= \frac{1}{2}\left(4mr^2\right)\dot{\theta}^2$			
	$= 2mr^2\dot{\theta}^2$	(M1)		Attempt at one energy term
	KE of particle $= \frac{1}{2}mv^2$			
	$= \frac{1}{2}(2m)\left(r\dot{\theta}\right)^2$			
	$= mr^2\dot{\theta}^2$			
	PE of particle $= mgh$			
	$= 2mgr\theta$	(A2,1,0)		Energy terms correct
	Conservation of energy			
	$2mr^2\dot{\theta}^2 + mr^2\dot{\theta}^2 = 2mgr\theta$			
	$3r\dot{\theta}^2 = 2g\theta$	(M1)		Form equation
	Differentiating $6r\dot{\theta}\ddot{\theta} = 2g\dot{\theta}$	(A1)		Correct differentiation
	$\Rightarrow 6r\ddot{\theta} = 2g$			
	$\ddot{\theta} = \dfrac{g}{3r}$	(A1)		Printed answer
(b)	$\therefore T = 4mr\left(\dfrac{g}{3r}\right)$			
	$= \dfrac{4mg}{3}$	B1	1	Substitute - must cancel r
	Total		**7**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	Use of I = $\dfrac{\int xy\,dx}{\int y\,dx}$	M1		[$\rho = 1$ allowed throughout] Stated or used
	$\int y \ dx = \text{semi circle area} = \frac{1}{2}\pi r^2$	A1		
	Use of diagram / by symmetry, $\bar{x} = 2\text{I}$	A1		Explains need for $2 \times$ Integral
	$\frac{1}{2}\pi r^2 \bar{x} = \int_o^r 2x\sqrt{r^2 - x^2}\,dx$	A1	4	Use of $y = \sqrt{r^2 - x^2}$ and rearrangement
	Alternative			
	Mass of elemental strip $= 2y\,\delta x\,\rho$			[Allow $\rho = 1$ throughout]
	Moment of elemental strip $= 2y\,\delta x\,\rho\,x$	(M1)		Elemental strip identified
	Total moment = $\rho\int_0^r 2xy\,dx$	(A1)		Integral formed
	Use of $\sum(mx) = \left(\sum m\right)\bar{x}$			
	Gives $\rho\int_0^r 2xy \ dy = \frac{1}{2}\rho\pi \ r^2\bar{x}$	(M1)		Equation formed
	$\Rightarrow \int_0^r 2x\sqrt{r^2 - x^2} \ dx = \frac{1}{2}\pi r^2\bar{x}$	(A1)		Use of $\sqrt{r^2 - x^2} = y$ and $\frac{1}{2}\pi r^2$
(ii)	$\int_0^r 2x\sqrt{r^2 - x^2} \ dx = \left[-\frac{2}{3}\left(r^2 - x^2\right)^{\frac{3}{2}}\right]_0^r$	M1		Attempt to integrate – inspection or substitution
		A1		Integrated correctly and limits substituted – condone sign error
	$= (0) - \left(-\frac{2}{3}r^3\right)$			
	$= \frac{2}{3}r^3$			
	$\therefore \frac{1}{2}\pi r^2\bar{x} = \frac{2}{3}r^3$			
	$\bar{x} = \frac{4r}{3\pi}$	A1	3	Printed answer
(b)(i)	$\frac{2}{3}(1.2) = 0.8 \text{ m}$	B1	1	
(ii)	$1.2 + \frac{4(0.5)}{3\pi} = 1.41 \text{ m}$	B1	1	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(iii)	Using $\left(\sum m\right)\bar{x} = \sum(mx)$			
	$\left[\frac{1}{2}(1)(1.2)\right][0.8] + \left[\frac{1}{2}\pi(0.5)^2\right][1.41]$			
	$= \left[\frac{1}{2}(1)(1.2) + \frac{1}{2}\pi(0.5)^2\right]\bar{x}$	M1		Form equation – Follow through (b)(i)
		A1√		One term correct
	$0.48 + 0.553\ldots = 0.992\ldots\bar{x}$	A1√		All terms correct
	$\therefore \bar{x} = 1.04$ m	A1	4	CAO
(c)				
	$\tan\theta = \frac{(1.2 - 1.04)}{0.5} = 0.32$	M1		$\tan\theta$ seen
		A1		Ratio correct structure = $\frac{1.2 - \bar{x}}{0.5}$
	$\theta = 18^\circ$	A1√	3	ft ratio error – correct use of $\tan^{-1}(\text{ratio})$
	Total		**16**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Rod *BC* has mass *m* Length $2l$			
	$I_G = \frac{1}{3}ml^2$	B1		
	(*G* is c of m for rod) $AG = l\sqrt{5}$	B1		Or $AG^2 = 5l^2$
	$I_{BC} = \frac{1}{3}ml^2 + m\left(l\sqrt{5}\right)^2$	M1		Use of parallel axis theorem
	$= \frac{16ml^2}{3}$	A1	4	
(b)	$I_{AD} = I_{AB} = \frac{4}{3}ml^2$	B1		Standard result
	$I_{DC} = I_{BC} = \frac{16ml^2}{3}$	B1		Use of part (a)
	$I_{SYSTEM} = \frac{4}{3}ml^2 + \frac{4}{3}ml^2 + \frac{16ml^2}{3} + \frac{16ml^3}{3}$	M1		4 rods or 3 particles considered
	$+3m\left(2l\right)^2 + 2m\left(8l^2\right) + m\left(2l\right)^2$	A2,1,0		–1 each type of error
	$= \frac{136ml^2}{3}$	A1	6	Printed answer
(c)	Gain in KE $= \frac{1}{2}I\dot{\theta}^2$ $= \frac{1}{2}\left(\frac{136}{3}\right)ml^2\dot{\theta}^2$	B1		Use of $\frac{1}{2}I\dot{\theta}^2 \left[\text{or} \frac{1}{2}I\omega^2\right]$
	Loss in PE for framework (using c. of mass) $= 4mg\left(2l\right)$	M1 A1		Use of *mgh* seen Correct loss of PE for framework only
	For particles PE lost $= 2mg\left(4l\right) + 3mg\left(4l\right)$	M1		$\sum$ Loss of PE for particles
	Total loss in PE = $28mgl$	A1		Correct total loss for particles
	$\frac{68}{3}ml^2\dot{\theta}^2 = 28mgl$	M1		Use of conservation of energy
	$\therefore \dot{\theta}^2 = \frac{3 \times 28 mgl}{68 ml^2} = \frac{21g}{17l}$ $\therefore \dot{\theta} = \sqrt{\frac{21g}{17l}}$	A1√	7	Their equation correctly rearranged for $\dot{\theta}^2$ or $\dot{\theta}$ Follow through one error

MM04 (cont)

Q	Solution	Marks	Total	Comments
7(c)	**Alternative**			
	Gain in KE = $\frac{1}{2}\left(\frac{136}{3}\right)ml^2\dot{\theta}^2$	(B1)		
	c. of mass for system			
	$2ml + 6m(2l) = 14m\bar{y}$	(M1)		Or by symmetry
	$\bar{y} = l$	(A1)		
	Loss in PE of system = mgh	(M1)		Use of mgh
	$= 14mg(2l) = 28mgl$	(A1√)		Follow through centre of mass error
	C of E $\frac{68}{3}ml^2\dot{\theta}^2 = 28mgl$	(M1)		
	$\dot{\theta}^2 = \frac{21g}{17l}$ $\dot{\theta} = \sqrt{\frac{21g}{17l}}$	(A1√)		Follow through one error
	Total		**17**	
	TOTAL		**75**	