Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM03

Unit Mechanics 3

Friday 22 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

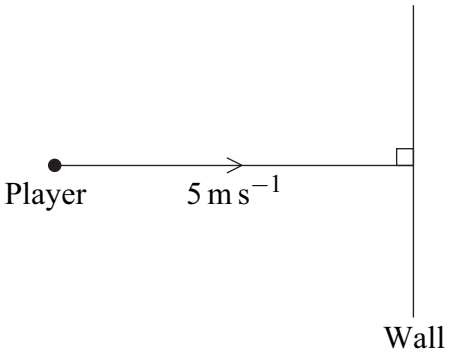
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 An ice-hockey player has mass 60 kg. He slides in a straight line at a constant speed of $5\,\text{m}\,\text{s}^{-1}$ on the horizontal smooth surface of an ice rink towards the vertical perimeter wall of the rink, as shown in the diagram.



The player collides directly with the wall, and remains in contact with the wall for 0.5 seconds.

At time t seconds after coming into contact with the wall, the force exerted by the wall on the player is $4 \times 10^4 t^2 (1 - 2t)$ newtons, where $0 \leqslant t \leqslant 0.5$.

(a) Find the magnitude of the impulse exerted by the wall on the player. *(4 marks)*

(b) The player rebounds from the wall. Find the player's speed immediately after the collision. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 A pile driver of mass m_1 falls from a height h onto a pile of mass m_2, driving the pile a distance s into the ground. The pile driver remains in contact with the pile after the impact. A resistance force R opposes the motion of the pile into the ground.

Elizabeth finds an expression for R as

$$R = \frac{g}{s}\left[s(m_1 + m_2) + \frac{h(m_1)^2}{m_1 + m_2}\right]$$

where g is the acceleration due to gravity.

Determine whether the expression is dimensionally consistent. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

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Answer space for question 2

3 **(In this question, take $g = 10\,\text{m}\,\text{s}^{-2}$.)**

A projectile is fired from a point O with speed u at an angle of elevation α above the horizontal so as to pass through a point P. The projectile travels in a vertical plane through O and P. The point P is at a horizontal distance $2k$ from O and at a vertical distance k above O.

(a) Show that α satisfies the equation

$$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0 \qquad \textit{(7 marks)}$$

(b) Deduce that

$$u^4 - 20ku^2 - 400k^2 \geqslant 0 \qquad \textit{(3 marks)}$$

QUESTION PART REFERENCE

Answer space for question 3

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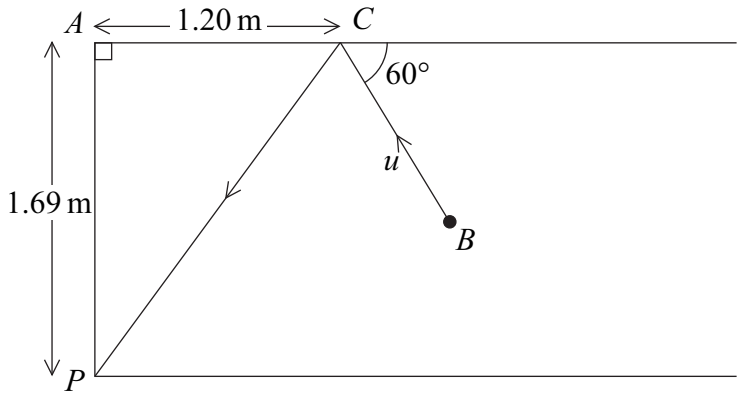
Answer space for question 3

4 The diagram shows part of a horizontal snooker table of width 1.69 m.

A player strikes the ball B directly, and it moves in a straight line. The ball hits the cushion of the table at C before rebounding and moving to the pocket at P at the corner of the table, as shown in the diagram. The point C is 1.20 m from the corner A of the table. The ball has mass 0.15 kg and, immediately before the collision with the cushion, it has velocity u in a direction inclined at 60° to the cushion. The **table** and the **cushion** are modelled as smooth.



(a) Find the coefficient of restitution between the ball and the cushion. *(5 marks)*

(b) Show that the magnitude of the impulse on the cushion at C is approximately $0.236u$. *(4 marks)*

(c) Find, in terms of u, the time taken between the ball hitting the cushion at C and entering the pocket at P. *(3 marks)*

(d) Explain how you have used the assumption that the cushion is smooth in your answers. *(1 mark)*

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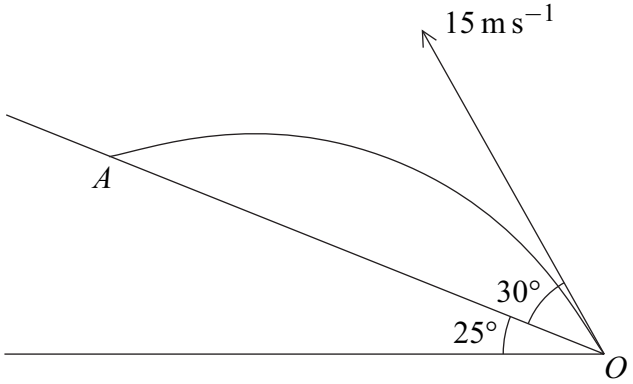
5 A particle is projected from a point O on a smooth plane, which is inclined at 25° to the horizontal. The particle is projected up the plane with velocity $15\,\mathrm{m\,s^{-1}}$ at an angle 30° above the plane. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Find the time taken by the particle to travel from O to A. *(4 marks)*

(b) The coefficient of restitution between the particle and the inclined plane is $\frac{2}{3}$.

Find the speed of the particle as it rebounds from the inclined plane at A. *(8 marks)*

QUESTION PART REFERENCE

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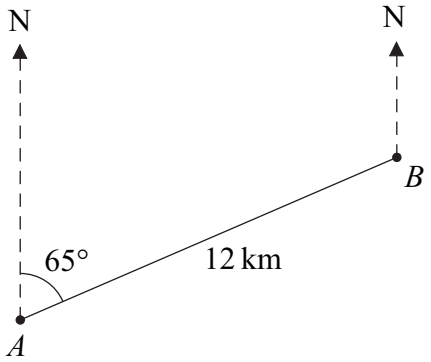
6 At noon, two ships, A and B, are a distance of 12 km apart, with B on a bearing of 065° from A. The ship B travels due north at a constant speed of $10\,\text{km}\,\text{h}^{-1}$. The ship A travels at a constant speed of $18\,\text{km}\,\text{h}^{-1}$.



(a) Find the direction in which A should travel in order to intercept B. Give your answer as a bearing. *(4 marks)*

(b) In fact, the ship A actually travels on a bearing of 065°.

(i) Find the distance between the ships when they are closest together. *(7 marks)*

(ii) Find the time when the ships are closest together. *(3 marks)*

QUESTION PART REFERENCE

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7 Two smooth spheres, A and B, have equal radii and masses $2m$ kg and m kg respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it collides with the sphere B, which has velocity $(2\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$. Immediately after the collision, the velocity of the sphere B is $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Show that the impulse exerted on B in the collision is $(6m\mathbf{j})\,\text{Ns}$. *(3 marks)*

(c) Find the coefficient of restitution between the two spheres. *(4 marks)*

(d) After the collision, each sphere moves in a straight line with constant speed. Given that the radius of each sphere is 0.05 m, find the time taken, from the collision, until the centres of the spheres are 1.10 m apart. *(5 marks)*

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END OF QUESTIONS

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