A-level
Mathematics

MFP4 Further Pure 4
Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	det $\mathbf{M} = -51$	**B1**		Correct determinant of $\mathbf{M}$ seen or used
	det $\mathbf{N} = \frac{1}{17}$	**M1**		Correct determinant of $\mathbf{N}$
	Hence det $\mathbf{MN}^2$ = det $\mathbf{M}$ x det $\mathbf{N}$ x det $\mathbf{N}$ $= \pm 51 \times \frac{1}{17} \times \frac{1}{17} = \pm 51 \times \frac{1}{289} = \frac{\pm 3}{17}$	**dM1**		Finds the scale factor of the enlargement **–** using determinant rules **–** follow through **their** $\lvert\mathbf{M}\rvert$.
	Volume of $S' = = 1.5(\text{cm}^3)$	**A1**	**4**	**CSO –** must be positive (–1.5 = 1.5 is A0)
	Total		**4**	

Q2	Solution	Mark	Total	Comment
	$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Since **B** represents a reflection then **B** = $\mathbf{B}^{-1}$ $\mathbf{A} = \mathbf{B}^{-1}\mathbf{C} =$ $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.28 & -0.96 & 0 \\ -0.96 & -0.28 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.96 & -0.28 & 0 \\ 0.28 & -0.96 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Transformation is a rotation about the z axis where $\cos\theta = -0.96$ or $\sin\theta = 0.28$ **A** represents a rotation of 164^0 about the z axis	B1 M1 A1 dM1 A1	5	States the correct matrix for **B** Pre-multiplies **C** by **their** $\mathbf{B}^{-1}$ or **B** in correct order. Can be $\mathbf{A} = \mathbf{B}^{-1}\mathbf{C}$ Correct matrix for **A** obtained Identifies **A** as a being a rotation about the z axis **and** attempts to find angle by matching $\cos\theta$ **or** $\sin\theta$ PI -163.7^o or AWRT 164^o Fully correct geometrical description stated – **CSO**
	Alternative If $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ Correct Answer = (B1)+M1A1dM1A1 Wrong Answer – but "rotation z axis" **with** either $\cos\theta = -0.96$ **or** $\sin\theta = 0.28$ = (B1)+SCB2 Wrong Answer – less than above = (B1)+M0A0dM0A0			
	Total		5	

Note:

$$\mathbf{CB} = \begin{pmatrix} 0.28 & -0.96 & 0 \\ -0.96 & -0.28 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -0.96 & 0.28 & 0 \\ -0.28 & -0.96 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \text{M0A0dM0A0}$$

$$\begin{pmatrix} \text{Gives} \\ \theta = 196.26 \end{pmatrix}$$

Q3	Solution	Mark	Total	Comment
(a)	determinant of **P** = $7k+14$	B1	1	Correct expression obtained
(b)	$\begin{bmatrix} -k-2 & -6 & -k+4 \\ -k-2 & 1 & -k-3 \\ 0 & -14 & -7k \end{bmatrix}$	M1 A(2,1,0)		**M1** one full row or column correct. **A1** at least 6 entries correct. **A2** all entries correct.
	$\mathbf{P}^{-1} = \dfrac{1}{7k+14}\begin{bmatrix} -k-2 & -k-2 & 0 \\ -6 & 1 & -14 \\ -k+4 & -k-3 & -7k \end{bmatrix}$	dM1		Transposition of **their** cofactors (with one further error at most) and dividing by **their** determinant
		A1	5	Fully correct - **CAO**
	Total		**6**	

Q4	Solution	Mark	Total	Comment
	$\mathbf{n} = \begin{pmatrix} 2 \\ p \\ 1 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$	B1		$\mathbf{n}, \mathbf{d}$ correctly identified **or** used
	$\mathbf{n}.\mathbf{d} = 4 - p$ $\lvert\mathbf{n}\rvert = \sqrt{p^2+5}$ $\lvert\mathbf{d}\rvert = \sqrt{6}$ angle between $\mathbf{n}$ and $\mathbf{d}$ is 60°	M1		Calculates $\mathbf{n}.\mathbf{d}$ correctly – follow through only **one** error in $\mathbf{d}$ only – $\mathbf{n}$ must be correct
	Hence $\dfrac{4-p}{\sqrt{6}\sqrt{p^2+5}} = k$ OE	dM1		Forms correct scalar product equation – follow through only **one** error in $\mathbf{d}$
	$k = \dfrac{1}{2}$	B1		$\dfrac{1}{2}$ coming from use of $\cos(60)$ or $\sin(30)$
	Squaring and rearranging gives $p^2 + 16p - 17 = 0$	dM1		Obtains a three term quadratic equation
	Hence $p = 1$ or $p = -17$	A1	6	Both values correct
	Total		**6**	
	Alternative $\mathbf{n} = \begin{pmatrix} 2 \\ p \\ 1 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$	(B1)		$\mathbf{n}, \mathbf{d}$ correctly identified **or** used
	$\mathbf{n} \times \mathbf{p} = \begin{pmatrix} 2p+1 \\ -3 \\ -2-p \end{pmatrix}$	(M1)		Calculates $\mathbf{n} \times \mathbf{p}$ correctly – follow through only **one** error in $\mathbf{d}$ only – $\mathbf{n}$ must be correct
	$\sqrt{5p^2+8p+14} = \sqrt{6}\sqrt{5+p^2} \times k$	(dM1)		Forms correct vector product equation – follow through only **one** error in $\mathbf{d}$
	$k = \dfrac{\sqrt{3}}{2}$	(B1)		$\dfrac{\sqrt{3}}{2}$ coming from use of $\sin(60)$
	Squaring and rearranging gives $p^2 + 16p - 17 = 0$	(dM1)		Obtains a three term quadratic equation
	Hence $p = 1$ or $p = -17$	(A1)	(6)	Both values correct

Q5	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix} 2 & 3 & -1 & 8 \\ 3 & k & 1 & -6 \\ 4 & 6 & k+1 & -4k+4 \end{bmatrix}$ r_3 replaced by $r_3 - 2r_1$ r_2 replaced by $r_2 - 1.5r_1$ $\begin{bmatrix} 2 & 3 & -1 & 8 \\ 0 & k-4.5 & 2.5 & -18 \\ 0 & 0 & k+3 & -4k-12 \end{bmatrix}$	M1		Alt 1 r_1 replaced by $2r_1 - r_3$ r_2 replaced by $4r_2 - 3r_3$ $\begin{bmatrix} 0 & 0 & -k-3 & 4k+12 \\ 0 & 4k-18 & 1-3k & 12k-36 \\ 4 & 6 & k+1 & -4k+4 \end{bmatrix}$ Alt 2 $r_2 + r_1$ $r_3 + (k+1)r_1$ $\begin{bmatrix} 2 & 3 & -1 & 8 \\ 5 & 3+k & 0 & 2 \\ 6+2k & 9+3k & 0 & 4k+12 \end{bmatrix}$ $r_3 - 3r_2$ $\begin{bmatrix} 2 & 3 & -1 & 8 \\ 5 & 3+k & 0 & 2 \\ -9+2k & 0 & 0 & 4k+6 \end{bmatrix}$ Attempts to eliminate variables to obtain an expression for one variable.
	Hence $z = -4$ or $\dfrac{-4k-12}{k+3}$ or $\dfrac{-4(k+3)}{k+3}$	A1		Correct answer (terms collected) for first variable
	Substituting $y(k-4.5) - 10 = -18$	M1		Attempts to substitute **and** rearranges to find expression for second variable
	$y = \dfrac{-16}{2k-9}$ or $\dfrac{-8}{k-4.5}$	A1		Correct answer (terms collected) for second variable
	Substituting $2x - \dfrac{48}{2k-9} + 4 = 8$ $x = \dfrac{4k+6}{2k-9}$ or $\dfrac{2k+3}{k-4.5}$ or $2 + \dfrac{24}{2k-9}$ or $2 + \dfrac{12}{k-4.5}$	A1	5	Correct answer (terms collected) for third variable

(a)	Alternative $\mathbf{M}^{-1} = \frac{1}{2k^2-3k-27}\begin{pmatrix} k^2+k-6 & -3k-9 & 3+k \\ 1-3k & 2k+6 & -5 \\ 18-4k & 0 & 2k-9 \end{pmatrix}$	**(M1)**		One **row** correct **with** attempt at $\frac{1}{\det}$
	$\mathbf{M}^{-1}\begin{pmatrix} 8 \\ -6 \\ -4k+4 \end{pmatrix}$			
	$= \frac{1}{2k^2-3k-27}\begin{pmatrix} 8k^2+8k-48+18k+54-4k^2-8k+12 \\ 8-24k-12k-36+20k-20 \\ 144-32k-8k^2+44k-36 \end{pmatrix}$	**(dM1)**		Two **rows** correct (unsimplified) **with** attempt at $\frac{1}{\det}$
	$= \frac{1}{2k^2-3k-27}\begin{pmatrix} 4k^2+18k+18 \\ -16k-48 \\ -8k^2+12k+108 \end{pmatrix}$	**(A1)** **(A1)** **(A1)**	**(5)**	A1 for each correct row **including** correct $\frac{1}{\det}$
	$\left\{= \frac{1}{(2k-9)(k+3)}\begin{pmatrix} 2(2k+3)(k+3) \\ -16(k+3) \\ -4(2k-9)(k+3) \end{pmatrix}\right\}$			

(b)	Sets **their** determinant = 0	**M1**		**MUST** be three term quadratic
	$(0 = 2k^2-3k-27)$ $k=-3$ or $k=4.5$	**A1,A1**	**3**	**A1** for each correct value
	Alternative Considers when denominators in expressions above are zero.	**(M1)**		
	$k=-3$ or $k=4.5$	**(A1,A1)**	**(3)**	

(c)	When k = 4.5, matrix becomes $\begin{bmatrix} 2 & 3 & -1 & 8 \\ 0 & 0 & 2.5 & -18 \\ 0 & 0 & 7.5 & -30 \end{bmatrix}$ OE			$\begin{matrix} 2x & +3y & -z & =8 \\ 3x & +\frac{9}{2}y & +z & =-6 \\ 4x & +6y & +\frac{11}{2}z & =-14 \end{matrix}$
	(Inconsistent,) therefore no solutions.	**M1**		k must be correct, with correct working.
	Three planes form a (triangular) prism.	**A1**		If “Three planes form a (triangular) prism.” With no working M0A0
	When k = –3, matrix becomes $\begin{matrix} 2x & +3y & -z & =8 \\ 3x & -3y & +z & =-6 \\ 4x & +6y & -2z & =16 \end{matrix}$ OE			
	(Consistent,) therefore infinite number of solutions.	**B1**		With correct working
	Two planes are identical **and** intersect the third plane (in a line). (Accept sheaf)	**B1**	**4**	With correct working **MUST** say two planes are the same **NOT** simply “sheaf” or “line”
	Total		**12**	

Q6	Solution	Mark	Total	Comment
(a)	$\mathbf{a}\times\mathbf{c}\cdot\mathbf{d}=$ $\begin{vmatrix} 2 & 2 & 1 \\ -1 & 3 & 4 \\ 6 & 4 & -3 \end{vmatrix}$			Correctly evaluates an appropriate triple scalar product (eg $\overrightarrow{AB}$, $\overrightarrow{AC}$ & $\overrightarrow{AD}$) **Or** uses correct vector product result in scalar product
	$\left.\begin{matrix} 2(-9-16)-2(3-24)+(-4-18) \\ \text{or} \\ 2(-9-16)+(-6-4)+6(8-3) \end{matrix}\right\}\text{OE}$ $=-30$	**M1**		$\begin{pmatrix}1\\4\\-3\end{pmatrix}\cdot\begin{pmatrix}-22\\4\\8\end{pmatrix}$ or $\begin{pmatrix}2\\-1\\6\end{pmatrix}\cdot\begin{pmatrix}-25\\10\\5\end{pmatrix}$ or $\begin{pmatrix}2\\3\\4\end{pmatrix}\cdot\begin{pmatrix}-21\\12\\9\end{pmatrix}$
	Volume = 30 (cubic units)	**A1**	**2**	States correct positive volume. (Condone –30 = 30)
(b)(i)				
	$\mathbf{v}$ or $\mathbf{w}=\overrightarrow{CB}=\begin{pmatrix}2\\-1\\6\end{pmatrix}$ or $\overrightarrow{CD}=\begin{pmatrix}-1\\1\\-7\end{pmatrix}=\overrightarrow{BE}$ or $\overrightarrow{CE}=\begin{pmatrix}1\\0\\-1\end{pmatrix}$ or $\overrightarrow{DB}=\begin{pmatrix}3\\-2\\13\end{pmatrix}$	**B1** **B1**		Any correct direction vector Second correct direction vector
	$\mathbf{u}=\begin{pmatrix}4\\2\\10\end{pmatrix}$ or $\begin{pmatrix}2\\3\\4\end{pmatrix}$ or $\begin{pmatrix}1\\4\\-3\end{pmatrix}$ or $\begin{pmatrix}3\\3\\3\end{pmatrix}$			
	Hence $\mathbf{r}=\begin{pmatrix}4\\2\\10\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\-7\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\6\end{pmatrix}$	**B1**	**3**	**OE** – Fully correct – there are many possible combinations

(b)(ii)	For plane $ABGD$, a perpendicular to the plane is $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \times \begin{pmatrix} -1 \\ 5 \\ -9 \end{pmatrix}$	**M1**		Uses two correct vectors in a vector product to find a normal to the plane Also $\overrightarrow{DB} = \begin{pmatrix} 3 \\ -2 \\ 13 \end{pmatrix}$
	$= \begin{pmatrix} -47 \\ 14 \\ 13 \end{pmatrix}$	**A1**		$\mathbf{n}$ correctly identified
	$k = \begin{pmatrix} -47 \\ 14 \\ 13 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 10 \end{pmatrix} = -30$			Other possible points $\begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix}$
	Hence equation is $\mathbf{r} \cdot \begin{pmatrix} -47 \\ 14 \\ 13 \end{pmatrix} = -30$ OE	**A1**	**3**	Uses scalar product to find the value of k **and** expresses plane in required form
(b)(iii)	Direction vector of line is parallel to $\overrightarrow{BD}$, hence $\mathbf{q} = \begin{pmatrix} -3 \\ 2 \\ -13 \end{pmatrix}$	**B1**		Finds a direction vector
	$\mathbf{p}$ = Position vector of a point on both planes = $\begin{pmatrix} 4 \\ 2 \\ 10 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$ most likely	**B1**		Other possibilities for point $\begin{pmatrix} 0 \\ 14/3 \\ -22/3 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ 23 \end{pmatrix}, \begin{pmatrix} 22/13 \\ 46/13 \\ 0 \end{pmatrix}$ Clearly identifies a common point
	Hence $\left[\mathbf{r} - \begin{pmatrix} 4 \\ 2 \\ 10 \end{pmatrix}\right] \times \begin{pmatrix} -3 \\ 2 \\ -13 \end{pmatrix} = \mathbf{0}$	**B1**	**3**	Fully correct form stated – including vector notation
	Total		**11**	

Q7	Solution	Mark	Total	Comment
(a)	Throughout this question: Condone missing brackets on factors, but penalise in final A1 CSO, even if recovered			Correct use of column/row operations to obtain first linear factor
	$\begin{vmatrix} x+y+z & y^2 & y+z \\ x+y+z & x^2 & x+z \\ x+y+z & 2y^2 & z \end{vmatrix}$	**M1**		$\begin{vmatrix} x-y & y^2-x^2 & y-x \\ y & x^2 & x+z \\ x+y & 2y^2 & z \end{vmatrix}$
	$= (x+y+z)\begin{vmatrix} 1 & y^2 & y+z \\ 1 & x^2 & x+z \\ 1 & 2y^2 & z \end{vmatrix}$	**A1**		$(y-x)\begin{vmatrix} -1 & y+x & 1 \\ y & x^2 & x+z \\ x+y & 2y^2 & z \end{vmatrix}$ Correct extraction of $x+y+z$ or $y-x$
	$= \left((x+y+z)\begin{vmatrix} 1 & y^2 & y+z \\ 0 & x^2-y^2 & x-y \\ 0 & y^2 & -y \end{vmatrix} \right)$			$\left((y-x)\begin{vmatrix} 0 & y+x & 1 \\ x+y+z & x^2 & x+z \\ x+y+z & 2y^2 & z \end{vmatrix} \right)$
	$\left((x+y+z)(x-y)\begin{vmatrix} 1 & y^2 & y+z \\ 0 & x+y & 1 \\ 0 & y^2 & -y \end{vmatrix} \right.$ or $\left. (x+y+z)y\begin{vmatrix} 1 & y^2 & y+z \\ 0 & x^2-y^2 & x-y \\ 0 & y & -1 \end{vmatrix} \right)$	**A1**		$(y-x)(x+y+z)\begin{vmatrix} 0 & y+x & 1 \\ 0 & x^2-2y^2 & x \\ 1 & 2y^2 & z \end{vmatrix}$ Correct extraction of second linear factor y, $x+y+z$ or $y-x$
	$=(x+y+z)(x-y)y\begin{vmatrix} 1 & y^2 & y+z \\ 0 & x+y & 1 \\ 0 & y & -1 \end{vmatrix}$	**A1**		Correct extraction of third linear factor y, $x+y+z$ or $y-x$
		dM1		Correctly expands final determinant (unsimplified) – **MUST** have scored at least M1A1A1
	$=-y(x+y+z)(x-y)(x+2y)$ OE	**A1**		Fully correct expansion (in linear form) – **CSO**
			6	

(b)	For a singular matrix det = 0 $-3(2+3+z)(2-3)(2+6)=0$ PI by $z+5=0$ Hence $z=\ -5$	**M1** **A1**	 **2**	Attempt to substitute in given values for $x(=2)$ and $y(=3)$ **and** set **their** determinant equal to zero. Be generous. Correct value obtained **– MUST** have scored **6** marks in part (a) **CSO**
	Total		**8**	
(b)	Alternative $\begin{vmatrix} 2 & 9 & 3+z \\ 3 & 4 & 2+z \\ 5 & 18 & z \end{vmatrix}$ $0=2(4z-36-18z)-3(9z-54-18z)$ $+5(18+9z-12-4z)$ Hence $z=\ -5$	 **(M1)** **(A1)**	 **(2)**	Independent of (a) OE **CSO** – doesn't need **6** marks in (a)

Q8	Solution	Mark	Total	Comment
(a)	Value = 1	B1	1	CAO
(b)(i)	$-2a + 3 = -5$ $-2b + c = 4$ $ac - 3b = 1$	M1		Forms **two** correct equations
	$a = 4$ $b = -3$ $c = -2$	B1 A1 A1	4	Finds correct value of a Finds correct value of b Finds correct value of c
(b)(ii)	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ mx+2 \end{bmatrix} = \begin{bmatrix} 4x+3mx+6 \\ -3x-2mx-4 \end{bmatrix}$ $\left.\begin{array}{l} x' = "4"x + 3mx + 6 \\ y' = "-3"x + "-2"mx + "-4" \end{array}\right\}$	B1F		Obtains expressions for **their** x' and y' in x only $x' = 4x + 3mx + 3k$ $y' = -3x - 2mx - 2k$
	Invariant implies $y' = mx' + 2$ Therefore $-3x - 2y = m(4x + 3y) + 2$ or $-3x - 2mx - 4 = m(4x + 3mx + 6) + 2$	M1		Invariant implies $y' = mx' + k$ or $-3x - 2mx - 2k = m(4x + 3mx + 3k) + k$ Applies invariant line condition $(k \neq 0)$
	$3(m+1)^2 x + 6(m+1) = 0$ Therefore $m = -1$ so $y = -x + 2$ is the required line	A1	3	$3(m+1)^2 x + 3k(m+1) = 0$ Obtains the correct equation for the invariant line through the point (0 , 2)
	Total		**8**	

Q9	Solution	Mark	Total	Comment
(a)	$\begin{vmatrix} -p-\lambda & q-p \\ p+q & p-\lambda \end{vmatrix} = 0$ $(-p-\lambda)(p-\lambda)-(p+q)(q-p)=0$	M1		Forms characteristic equation by expanding determinant. PI = 0
	$-p^2+\lambda^2+p^2-q^2=0$ $(\lambda-q)(\lambda+q)=0$ $\lambda=\pm q$	A1,A1	3	A1 each eigenvalue
(b)(i)	$\begin{bmatrix} -p & q-p \\ p+q & p \end{bmatrix}\begin{bmatrix} q-p \\ p+q \end{bmatrix}=\begin{bmatrix} q^2-pq \\ q^2+pq \end{bmatrix}=q\begin{bmatrix} q-p \\ p+q \end{bmatrix}$	M1		Must have correct intermediate step from LHS to RHS
	$\lambda = q$	A1	2	
(ii)	$\begin{bmatrix} -p+q & q-p \\ p+q & p+q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$			
	Top row gives $(-p+q)x+(q-p)y=0$ **or** Bottom row gives $(p+q)x+(p+q)y=0$	M1		Minimal acceptable: $-px+qx+qy-py=0$ or $px+qx+py+qy=0$
	Hence an eigenvector is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ which contains no p or q terms	A1	2	Obtains correct eigenvector **and** comments on there being no p, q terms
	ALTERNATIVE for (b)(i) $\lambda = q$ $\begin{bmatrix} -p-q & q-p \\ p+q & p-q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Using top row $(-p-q)x+(q-p)y=0$ **or** using second row $(p+q)x+(p-q)y=0$	(M1)		
	$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} q-p \\ p+q \end{bmatrix}$	(A1)	(2)	**Clear working** to show/verify eigenvector. Must state $\lambda = q$ somewhere. **AG** – **CSO** – Fully explained

(c)(i)	$\mathbf{D}=\begin{bmatrix} q & 0 \\ 0 & -q \end{bmatrix}$	B1F		**Their** matrix $\mathbf{D}$ stated or $\mathbf{D}=\begin{bmatrix} -q & 0 \\ 0 & q \end{bmatrix}$ & $\mathbf{U}=\begin{bmatrix} 1 & -p+q \\ -1 & p+q \end{bmatrix}$
	$\mathbf{U}=\begin{bmatrix} -p+q & 1 \\ p+q & -1 \end{bmatrix}$ or $\begin{bmatrix} -p+q & -1 \\ p+q & 1 \end{bmatrix}$	B1F	2	**Their** correct matrix $\mathbf{U}$ (if zero column **B0**) stated **MUST** match order in $\mathbf{D}$
(ii)	$\mathbf{U}^{-1}=$ $\frac{1}{-2q}\begin{bmatrix} -1 & -1 \\ -p-q & -p+q \end{bmatrix}$ or $\frac{1}{2q}\begin{bmatrix} 1 & 1 \\ -p-q & -p+q \end{bmatrix}$ or $\frac{1}{2q}\begin{bmatrix} 1 & 1 \\ p+q & p-q \end{bmatrix}$	B1		Finds $\mathbf{U}^{-1}$ correctly or $\frac{1}{2q}\begin{bmatrix} p+q & p-q \\ 1 & 1 \end{bmatrix}$ **CAO**
	$\left(\mathbf{M}^n=\mathbf{U}\mathbf{D}^n\mathbf{U}^{-1}\right)$ $\frac{1}{2q}\begin{bmatrix} -p+q & 1 \\ p+q & -1 \end{bmatrix}\begin{bmatrix} q^n & 0 \\ 0 & (-q)^n \end{bmatrix}\begin{bmatrix} 1 & 1 \\ p+q & p-q \end{bmatrix}$	M1		Multiplies **their** matrices in correct order with $\mathbf{D}^n$ as shown ($\mathbf{U}$ **MUST** be non-singular) or $\frac{1}{2q}\begin{bmatrix} 1 & -p+q \\ -1 & p+q \end{bmatrix}\begin{bmatrix} (-q)^n & 0 \\ 0 & q^n \end{bmatrix}\begin{bmatrix} p+q & p-q \\ 1 & 1 \end{bmatrix}$
	If n is an odd number then $(-q)^n=-q^n$ $\left(=\frac{1}{2q}\begin{bmatrix} -p+q & 1 \\ p+q & -1 \end{bmatrix}\begin{bmatrix} q^n & 0 \\ 0 & -q^n \end{bmatrix}\begin{bmatrix} 1 & 1 \\ p+q & p-q \end{bmatrix}\right.$ $\left.=\frac{q^n}{2q}\begin{bmatrix} -p+q & 1 \\ p+q & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ p+q & p-q \end{bmatrix}\right)$	A1		Explains the significance of n being odd and uses result
	$=\frac{q^{n-1}}{2}\begin{bmatrix} -p+q & -1 \\ p+q & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ p+q & p-q \end{bmatrix}$ or $=\frac{q^{n-1}}{2}\begin{bmatrix} -p+q & 1 \\ p+q & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -p-q & -p+q \end{bmatrix}$	dM1		or $=\frac{q^{n-1}}{2}\begin{bmatrix} 1 & -p+q \\ -1 & p+q \end{bmatrix}\begin{bmatrix} -p-q & -p+q \\ 1 & 1 \end{bmatrix}$ or $=\frac{q^{n-1}}{2}\begin{bmatrix} -1 & -p+q \\ 1 & p+q \end{bmatrix}\begin{bmatrix} p+q & p-q \\ 1 & 1 \end{bmatrix}$ Correctly multiplies a pair of matrices together
	$=\frac{q^{n-1}}{2}\begin{bmatrix} -2p & -2p+2q \\ 2p+2q & 2p \end{bmatrix}$ $=q^{n-1}\begin{bmatrix} -p & -p+q \\ p+q & p \end{bmatrix}$	A1		All matrices originally correct & multiplied together correctly to form one matrix.
	$=q^{n-1}\mathbf{M}$	A1	6	Completes proof to show result **AG**

	ALTERNATIVE for (c)(ii)			
	$\mathbf{M}^n = \mathbf{U}\mathbf{D}^n\mathbf{U}^{-1}$	**(M1)**		Seen **and** correctly used
	If n is an odd number then $(-q)^n = -q^n$	**(A1)**		Explains the significance of n being odd
	$\mathbf{D}^n = \begin{pmatrix} q^n & 0 \\ 0 & (-q)^n \end{pmatrix} = \begin{pmatrix} q^n & 0 \\ 0 & -q^n \end{pmatrix}$	**(A1)**		Uses the above result in $\mathbf{D}^n$
	$= q^{n-1}\begin{pmatrix} q & 0 \\ 0 & -q \end{pmatrix} = q^{n-1}\mathbf{D}$	**(A1)**		Expresses $\mathbf{D}^n$ in terms of $\mathbf{D}$
	$\therefore \mathbf{M}^n = \mathbf{U}\left(q^{n-1}\mathbf{D}\right)\mathbf{U}^{-1}$	**(dM1)**		Substitutes $q^{n-1}\mathbf{D}$ for $\mathbf{D}^n$
	$= q^{n-1}\mathbf{U}\mathbf{D}\mathbf{U}^{-1}$ $= q^{n-1}\mathbf{M}$	**(A1)**	**(6)**	Completes proof to show result **AG**
	Total		**15**	