Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates’ responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates’ scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates’ reactions to a particular paper. Assumptions about future mark schemes on the basis of one year’s document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	det **A** = $5p - 1$ det **B** = $p^2 - 10p - 11$	B1 M1A1	**3**	M1A0 if num error(s) made
(b)	Use of det(**AB**) = det **A** det **B** Finding three values of p $p = \frac{1}{5}, 11, -1$	B1 M1 A1F	**3**	PI Allow correct factors here ft numerical errors in (a)
	Total		**6**	
2	$\begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$ & $\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$	B1		used or written down
	Mult'n of these in the correct order	B1		at least two entries correct
	Use of addition formulae	M1		At least once
	$\begin{bmatrix} \cos(2\alpha+\beta) & \sin(2\alpha+\beta) \\ \sin(2\alpha+\beta) & -\cos(2\alpha+\beta) \end{bmatrix}$	A1F		ft only for use of clockwise rot'n and/or mult'n in wrong order
	Reflection ... ... in $y = x\tan\left(\alpha + \frac{1}{2}\beta\right)$	A1F A1F	**6**	ft as above ft as above
	Total		**6**	
3(a)	Vector product attempted	M1		
	$\mathbf{p} \times \mathbf{q} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \times \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 \\ 45 \\ -30 \end{bmatrix}$	A1		
	$\ldots = 15\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$, so $t = -2$	A1	**3**	
(b)	Scalar triple product attempted	M1		OE, eg determinant
	$\mathbf{p} \times \mathbf{q} \bullet \mathbf{r} = 15\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix} = 15(13 - 2t)$	A1		
	$\ldots = 0$, so $t = 6\frac{1}{2}$	A1	**3**	
	ALT: $5\mathbf{p} + \mathbf{q} = 6\mathbf{r}$ $\ldots \Rightarrow t = 6\frac{1}{2}$	B2,0 B1		or any correct linear relationship
	Total		**6**	

Q	Solution	Marks	Total	Comments
4(a)	$\begin{vmatrix} 2 & 1 & 3 \\ 5 & -2 & a+1 \\ a & 2 & 4 \end{vmatrix} = a^2 + 3a - 10$	M1 A1		Attempt at det of coeff matrix Correct (accept unsimplified)
	Equating to 0 and solving quadratic in a $a = 2, -5$	m1 A1	4	**SC: B1** for verifying $a = 2$ **B1** for verifying $a = -5$
(b)	$\begin{aligned} 2x + y + 3z &= 3 \\ 5x - 2y + 3z &= 3 \\ 2x + 2y + 4z &= b \end{aligned}$	B1		
	Eliminations leading to two equations in two variables	M1		
	Further elimination leading to value of b	m1		
	$b = 4$	A1	4	
	ALT: Finding two variables in terms of third	M1		eg $y = x$ and $z = 1 - x$
	Substituting into third equation	m1		
	$b = 4$	A1		
			8	
5(a)	**(i)** Characteristic eqn $\lambda^2 - 9\lambda + 14 = 0$ $\lambda = 2, 7$	M1A1 A1		M1A0 if num error(s) made
	Substituting back for at least one eval	m1		for $\lambda = 2$, $-x + 3y = 0$ or for $\lambda = 7$, $-2x + y = 0$
	evecs $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	A1A1	6	or non-zero multiples
	(ii) $\mathbf{U} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}$	B1FB1F		Columns of **U** and **D** are interchangeable, but must match; ft wrong answers in (i)
	$\mathbf{U}^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$	B1F B1F	4	1/det **U** adjoint matrix; ft incorrect **U** (provided det $\neq 0$)
(b)	**(i)** evals of $\mathbf{M}^3$ are λ^3, μ^3	B1		
	since $\mathbf{M}^3 = \mathbf{U}\,\mathbf{D}^3\,\mathbf{U}^{-1}$	E1	2	
	(ii) evecs of $\mathbf{M}^3$ are $\mathbf{v}_1$ and $\mathbf{v}_2$	B1	1	
			13	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mathbf{r} = \begin{bmatrix}1\\2\\3\end{bmatrix} + \lambda\begin{bmatrix}2\\3\\6\end{bmatrix}$	B2,1	**2**	Any correct vector line equation; B1 if one vector correct, or if both correct but equation not in correct form
	(ii) $\mathbf{r} = \begin{bmatrix}1 & 4 & -3\\2 & -1 & 0\\1 & 1 & -1\end{bmatrix}\begin{bmatrix}1+2\lambda\\2+3\lambda\\3+6\lambda\end{bmatrix} = \begin{bmatrix}-4\lambda\\\lambda\\-\lambda\end{bmatrix}$	M1 A1 A1		Attempt at multiplication At least one entry correct All three correct
	Clear and valid explanation that this is a line through O	E1	**4**	
(b) (i)	$\begin{bmatrix}1 & 4\\2 & -1\end{bmatrix}\begin{bmatrix}p\\\frac{1}{2}p+k\end{bmatrix} = \begin{bmatrix}3p+4k\\\frac{3}{2}p-k\end{bmatrix}$	B1 M1A1		For LHS For RHS
	Answer satisfies $y = \frac{1}{2}x - 3k$	A1	**4**	
	(ii) Equal gradients, hence parallel	E1F		ft if previous answer is of the form $y = \frac{1}{2}x + c$
	Distance = $\lvert k - c\rvert \cos\theta$ with $\tan\theta = \frac{1}{2}$	M1		Allow incorrect value of c here
	...$= \dfrac{8k}{\sqrt{5}}$	A1	**3**	Allow $3.58k$
			13	
7 (a)(i)	Appropriate row/column operation	M1		eg $R_1' = R_1 + R_3$, $R_3' = R_3 + R_1$ or $C_3' = C_3 - nC_2$
	$\Delta = \begin{vmatrix}n^2+n+1 & 0 & 0\\0 & 1 & n\\1 & -(n+1) & 1\end{vmatrix}$			
	... $= (n^2 + n + 1)\begin{vmatrix}1 & 0 & 0\\0 & 1 & n\\1 & -(n+1) & 1\end{vmatrix}$	A1	**2**	Factor correctly extracted
(ii)	Expanding remaining determinant	M1		OE
	$\Delta = (n^2 + n + 1)^2$	A1	**2**	
(b)	$\Delta = (n^2 + n)^2 + 2n^2 + 2n + 1$ $\ldots = (n^2 + n)^2 + (n + 1)^2 + n^2$	B1 B1	**2**	Accept unsimplified
(c)	Setting $n = 10$	M1		
	$111^2 = 12321 = 110^2 + 11^2 + 10^2$	A1	**2**	
			8	

Q	Solution	Marks	Total	Comments
8(a)	Use of sin or $\cos\theta = \dfrac{\text{scalar product}}{\text{product of moduli}}$	M1		using $\begin{bmatrix}3\\-2\\6\end{bmatrix}$ and $\begin{bmatrix}2\\1\\2\end{bmatrix}$
	Numerator = 16, denominator = 21	B1B1		Allow numerator $\sqrt{185}$
	$\sin\theta = \dfrac{16}{21} \Rightarrow \theta \approx 49.6^{o}$	A1	**4**	Allow AWRT 49.6
(b)	$\begin{bmatrix}2\lambda+1\\\lambda+2\\2\lambda-7\end{bmatrix} \bullet \begin{bmatrix}3\\-2\\6\end{bmatrix} = 37$	M1		
	$6\lambda + 3 - 2\lambda - 4 + 12\lambda - 42 = 37$	m1		with attempt to solve
	$\ldots \Rightarrow \lambda = 5$	A1		
	giving $P = (11, 7, 3)$	B1F	**4**	ft wrong value of λ
(c)(i)	Use of the vectors $\begin{bmatrix}3\\-2\\6\end{bmatrix}$ and $\begin{bmatrix}2\\1\\2\end{bmatrix}$	M1		
	Vector product attempted	m1		OE
	Required vector is $\begin{bmatrix}-10\\6\\7\end{bmatrix}$	A1	**3**	Or a non-zero multiple
(ii)	$\mathbf{a} = \begin{bmatrix}11\\7\\3\end{bmatrix}$	B1F		ft wrong answer in (b)
	$\mathbf{b} = \begin{bmatrix}-10\\6\\7\end{bmatrix} \times \begin{bmatrix}3\\-2\\6\end{bmatrix} = \begin{bmatrix}50\\81\\2\end{bmatrix}$	M1A1F		Or a non-zero multiple; ft wrong answer to (c)(i)
	Fully correct equation for L'	A1	**4**	
			15	
	TOTAL		**75**	