

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2013

Mathematics

MFP2

Unit Further Pure 2

Thursday 6 June 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 3 M F P 2 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

- 1 (a)** Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 6i| = 3 \quad (3 \text{ marks})$$

- (b)** It is given that z satisfies the equation $|z - 6i| = 3$.

- (i)** Write down the greatest possible value of $|z|$. (1 mark)

- (ii)** Find the greatest possible value of $\arg z$, giving your answer in the form $p\pi$, where $-1 < p \leq 1$. (3 marks)

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2 (a) (i) Sketch on the axes below the graphs of $y = \sinh x$ and $y = \cosh x$. (3 marks)

(ii) Use your graphs to explain why the equation

$$(k + \sinh x) \cosh x = 0$$

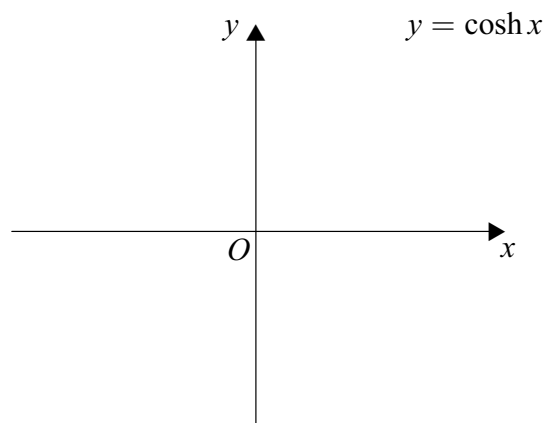
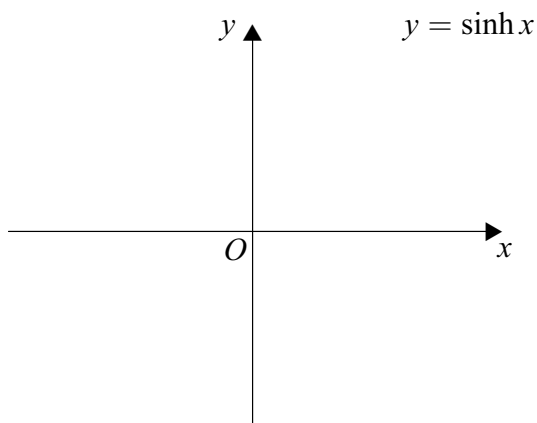
where k is a constant, has exactly one solution. (1 mark)

(b) A curve C has equation $y = 6 \sinh x + \cosh^2 x$. Show that C has only one stationary point and show that its y -coordinate is an integer. (5 marks)

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(a)(i)



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3 The sequence u_1, u_2, u_3, \dots is defined by

$$u_1 = 2, \quad u_{n+1} = \frac{5u_n - 3}{3u_n - 1}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{3n + 1}{3n - 1} \quad (6 \text{ marks})$$

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4 (a) Given that $f(r) = r^2(2r^2 - 1)$, show that

$$f(r) - f(r - 1) = (2r - 1)^3 \quad (3 \text{ marks})$$

(b) Use the method of differences to show that

$$\sum_{r=n+1}^{2n} (2r - 1)^3 = 3n^2(10n^2 - 1) \quad (4 \text{ marks})$$

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5 The cubic equation

$$z^3 + pz^2 + qz + 37 - 36i = 0$$

where p and q are constants, has three complex roots, α , β and γ .

It is given that $\beta = -2 + 3i$ and $\gamma = 1 + 2i$.

- (a) (i) Write down the value of $\alpha\beta\gamma$. *(1 mark)*
- (ii) Hence show that $(8 + i)\alpha = 37 - 36i$. *(2 marks)*
- (iii) Hence find α , giving your answer in the form $m + ni$, where m and n are integers. *(3 marks)*
- (b) Find the value of p . *(1 mark)*
- (c) Find the value of the complex number q . *(2 marks)*

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6 (a) Show that $\frac{1}{5 \cosh x - 3 \sinh x} = \frac{e^x}{m + e^{2x}}$, where m is an integer. (3 marks)

(b) Use the substitution $u = e^x$ to show that

$$\int_0^{\ln 2} \frac{1}{5 \cosh x - 3 \sinh x} dx = \frac{\pi}{8} - \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right) \quad (5 \text{ marks})$$

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7 (a) (i) Show that

$$\frac{d}{du} (2u\sqrt{1+4u^2} + \sinh^{-1} 2u) = k\sqrt{1+4u^2}$$

where k is an integer.

(4 marks)

(ii) Hence show that

$$\int_0^1 \sqrt{1+4u^2} du = p\sqrt{5} + q \sinh^{-1} 2$$

where p and q are rational numbers.

(2 marks)

(b) The arc of the curve with equation $y = \frac{1}{2} \cos 4x$ between the points where $x = 0$ and $x = \frac{\pi}{8}$ is rotated through 2π radians about the x -axis.

(i) Show that the area S of the curved surface formed is given by

$$S = \pi \int_0^{\frac{\pi}{8}} \cos 4x \sqrt{1+4 \sin^2 4x} dx$$

(2 marks)

(ii) Use the substitution $u = \sin 4x$ to find the exact value of S .

(4 marks)

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8 (a) (i) Use de Moivre's theorem to show that

$$\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$$

and find a similar expression for $\sin 4\theta$. (5 marks)

(ii) Deduce that

$$\tan 4\theta = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} \quad (3 \text{ marks})$$

(b) Explain why $t = \tan \frac{\pi}{16}$ is a root of the equation

$$t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$$

and write down the three other roots in trigonometric form. (4 marks)

(c) Hence show that

$$\tan^2 \frac{\pi}{16} + \tan^2 \frac{3\pi}{16} + \tan^2 \frac{5\pi}{16} + \tan^2 \frac{7\pi}{16} = 28 \quad (5 \text{ marks})$$

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END OF QUESTIONS

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