AQA

A-level
MATHEMATICS

Unit Decision 2

Wednesday 28 June 2017　　　Morning　　　Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer each question in the space provided for that question.
  If you require extra space, use an AQA supplementary answer book; do **not**
  use the space provided for a different question.
• Do not write outside the box around each page.
• Show all necessary working, otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to
  be marked.
• The **final** answer to questions requiring the use of calculators should be given to
  three significant figures, unless stated otherwise.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

MD02

Answer **all** questions.

Answer each question in the space provided for that question.

1 A student is solving a linear programming problem using the simplex method. The student obtains the following tableau.

P	x	y	z	r	s	t	value
1	0	4	0	3	0	2	20
0	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	6
0	0	$\frac{5}{2}$	1	1	0	$-\frac{3}{2}$	1
0	0	$\frac{3}{4}$	0	$-\frac{1}{4}$	1	$-\frac{1}{2}$	$\frac{1}{4}$

(a) Explain how you know that the tableau is optimal.

[1 mark]

(b) Interpret this tableau, stating the values of all of the variables.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

QUESTION PART REFERENCE	Answer space for question 1
QUESTION PART REFERENCE	Answer space for question 1

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2 Erica and Viggo play a zero-sum game. The game is represented by the following pay-off matrix for Erica.

Viggo

Erica

Strategy	W	X	Y	Z
A	4	–2	3	1
B	–1	–4	2	3
C	–2	0	–2	4
D	–3	–2	1	0

Find the play-safe strategies for Erica and Viggo and state, with a reason, whether or not the game has a stable solution.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	Answer space for question 2
QUESTION PART REFERENCE	Answer space for question 2

Turn over ▶

3 The table below shows the times taken, in minutes, by five people A, B, C, D and E to complete the tasks V, W, X, Y and Z. The time taken for person B to complete task W is x minutes.

	A	*B*	*C*	*D*	*E*
V	12	22	22	12	5
W	30	*x*	20	30	25
X	15	20	11	18	20
Y	27	25	30	30	20
Z	10	8	9	10	12

Using the Hungarian algorithm, each of the five tasks is to be given to a different one of the five people so that the total time for the completion of the five tasks is minimised.

In the case where $x > 25$, by reducing the **rows first**, find all the possible ways of allocating the five tasks to the five people so as to minimise the total time. State this minimum total time.

[9 marks]

Answer space for question 3

QUESTION PART REFERENCE	Answer space for question 3
QUESTION PART REFERENCE	Answer space for question 3

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IB/G/Jun17/MD02

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IB/G/Jun17/MD02

4 A major project has been divided into a number of activities, as shown in the table. The minimum time to complete each activity is also shown. The time taken to complete activity F is x days, where x is an integer.

Activity	Immediate predecessor	Duration (days)
A	-	4
B	-	6
C	-	5
D	-	12
E	A, B	8
F	B, C	x
G	C, D	7
H	E, F	16
I	F, G	11
J	H, I	4
K	J	6
L	J	4

(a) On the page opposite, construct an activity network for the project.

(Activity A has already been drawn for you.)

[2 marks]

(b) In the case where $x < 8$, find:

(i) the earliest start time and latest finish time for each activity;

(ii) the critical path(s).

[5 marks]

(c) In the case where $x > 8$, find, in terms of x:

(i) the minimum completion time for the project;

(ii) the float time for activity L.

[3 marks]

(d) In the case where $x = 5$, draw a Gantt chart on **Figure 1** on page 13.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

Earliest start time

Duration

Latest finish time

	A
	4

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 4
QUESTION PART REFERENCE	Answer space for question 4

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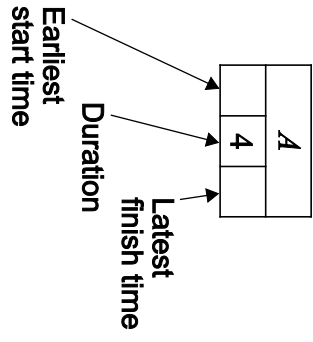
QUESTION PART REFERENCE	**Answer space for question 4**

Figure 1

Turn over ▶

5 Solve the following linear programming problem using the simplex method

Maximise $P = 2x - 3y + 5z$

subject to
$$3x - 2y + 2z \leqslant 44$$
$$4x + 2y - z \leqslant 44$$
$$5x + y - 4z \leqslant 44$$

and $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$

choosing the first pivot from the z-column.

[10 marks]

QUESTION PART REFERENCE | **Answer space for question 5**

QUESTION PART REFERENCE	Answer space for question 5

QUESTION PART REFERENCE	Answer space for question 5

Turn over ▶

6 Jeremy and Nigel play a zero-sum game. The game is represented by the following pay-off matrix for Jeremy.

Nigel

Strategy	D	E	F
A	3	1	−2
B	2	−3	−3
C	−1	2	4

Jeremy

Find the optimal mixed strategy for Jeremy and the value of the game **for Nigel**.

[8 marks]

Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

Turn over ▶

7   A courier is considering various routes to travel from S to T. In the network, the number on each edge is the expected profit, in pounds, for delivering parcels along a particular edge. A negative number represents a loss when the travelling costs exceed the income from parcels delivered along that edge.

By completing the table on the page opposite, use dynamic programming working backwards from T to find the courier's maximum profit. State the route(s) corresponding to this maximum profit.

[10 marks]

QUESTION PART REFERENCE	Answer space for question 7

Answer space for question 7

Stage	State	From	Calculation	Value
1	J	T		
	K	T		

Turn over ▶

8 The network shows a system of pipes with lower and upper capacities for each pipe in litres per second.
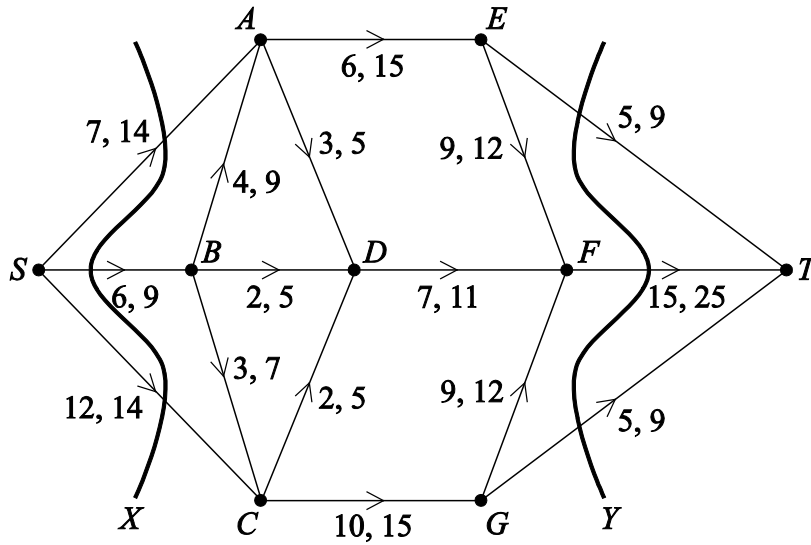
(a) Find the value of the cut:

(i) X;

(ii) Y.

[2 marks]

(b) (i) Explain why the flow along SB must be 9.

(ii) State the value of the flow along FT.

[2 marks]

(c) Complete **Figure 2** on the opposite page to show the minimum flow through the network.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

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QUESTION PART REFERENCE	**Answer space for question 8**

Figure 2
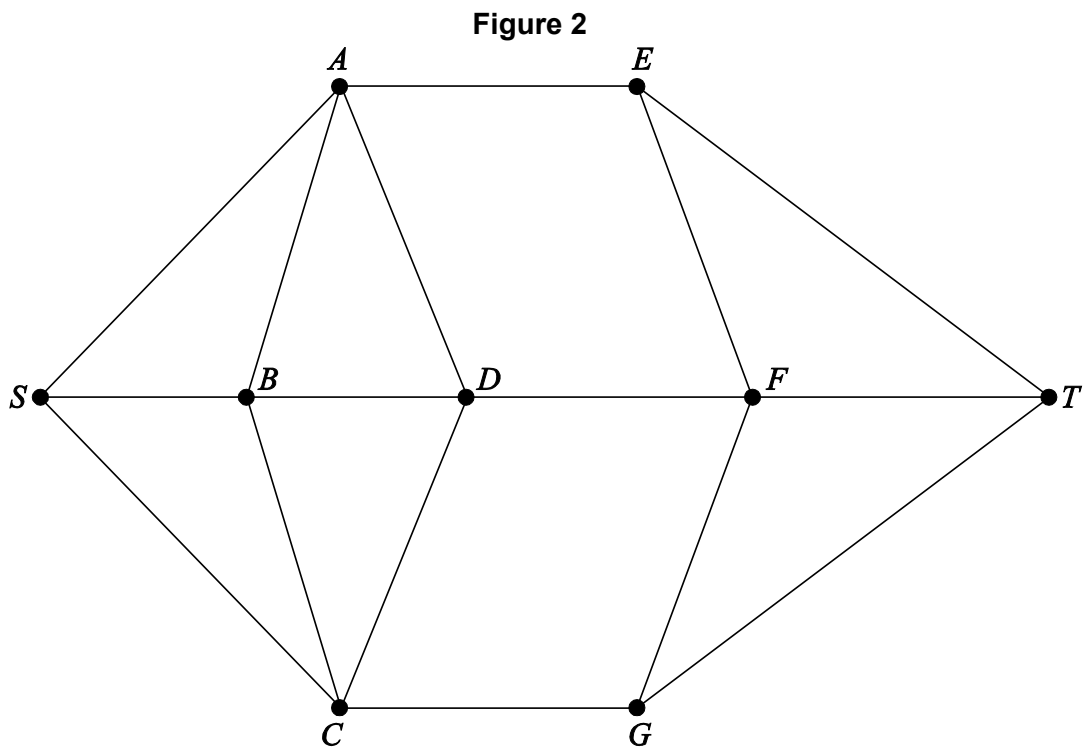
9 A company has warehouses at P and Q, and goods are to be transported to retail outlets at X, Y and Z. There are local depots at A, B, C, D and E.

The possible routes are shown in the diagram. The number on each edge represents the capacity of the edge, in van loads per week, and the numbers in the circles represent a possible flow of 58 van loads per week.
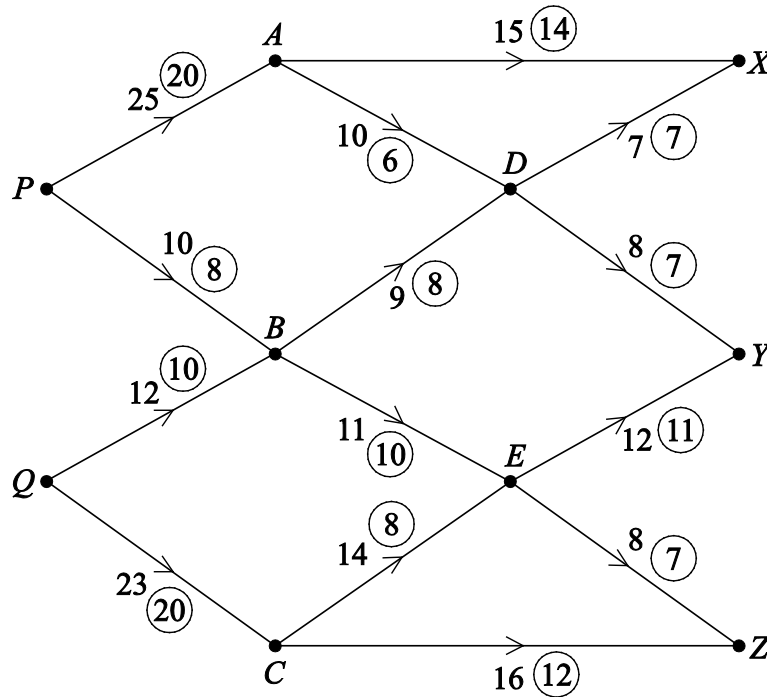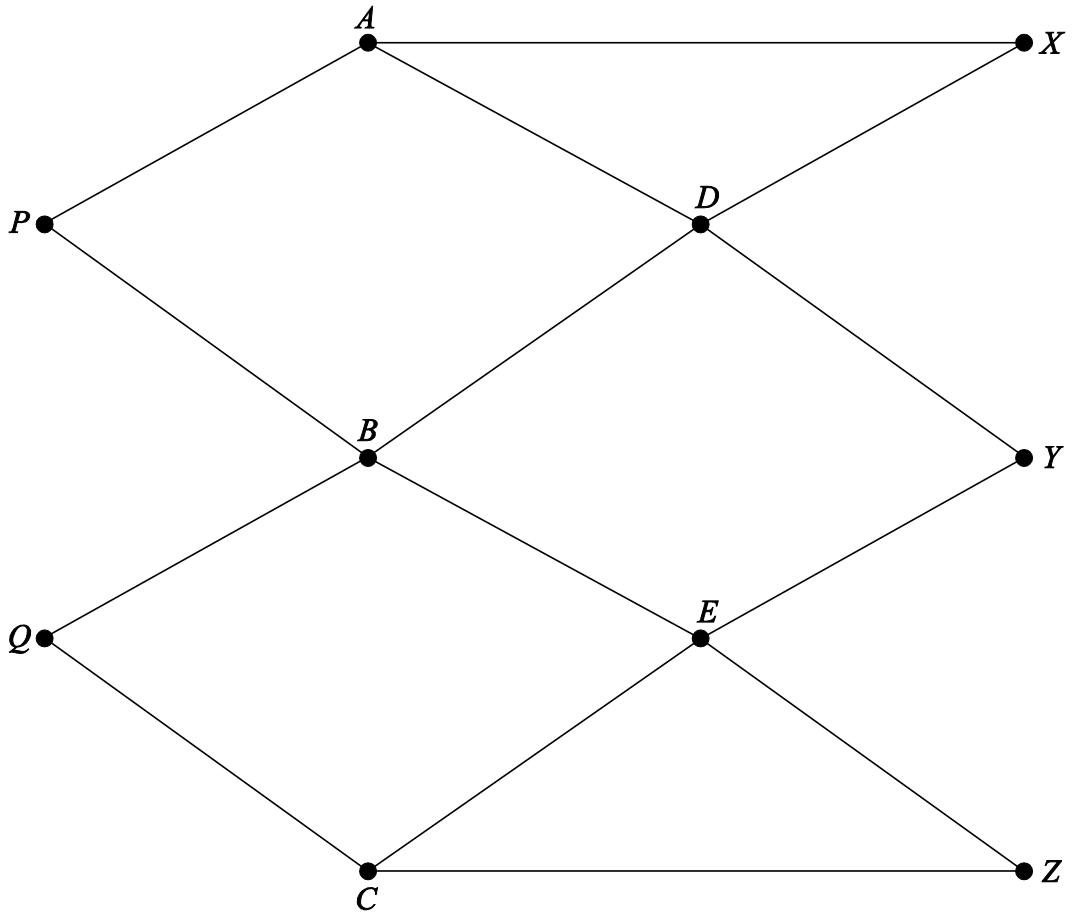

(a) On **Figure 3 opposite**, add a super-source S and a super-sink T, and appropriate edges so as to produce a directed network with a single source and a single sink.

[1 marks]

(b) On **Figure 3**, taking the given flow of 58 as the initial flow pattern, indicate the potential increases and decreases of flow along each edge.

[3 marks]

(c) Use flow augmentation on **Figure 3** to find the maximum flow from S to T. You must indicate any flow augmentation routes in the table and modify potential increases and decreases of the flow on the network.

[5 marks]

(d) State the maximum flow and confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 9

Figure 3

Route	Flow

Turn over ▶

QUESTION PART REFERENCE	**Answer space for question 9**
2	

END OF QUESTIONS