Decision 1 Travelling Salesman Questions

8 Salvadore is visiting six famous places in Barcelona: La Pedrera (L), Nou Camp (N), Olympic Village (O), Park Guell (P), Ramblas (R) and Sagrada Familia (S). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel.

The table shows the times, in minutes, that it will take to travel between the six places.

From \ To	**La Pedrera** (L)	**Nou Camp** (N)	**Olympic Village** (O)	**Park Guell** (P)	**Ramblas** (R)	**Sagrada Familia** (S)
La Pedrera (L)	—	35	30	30	37	35
Nou Camp (N)	25	—	20	21	25	40
Olympic Village (O)	15	40	—	25	30	29
Park Guell (P)	30	35	25	—	35	20
Ramblas (R)	20	30	17	25	—	25
Sagrada Familia (S)	25	35	29	20	30	—

(a) Find the total travelling time for:

(i) the route $LNOL$; *(1 mark)*

(ii) the route $LONL$. *(1 mark)*

(b) Give an example of a Hamiltonian cycle in the context of the above situation. *(1 mark)*

(c) Salvadore intends to travel from one place to another until he has visited all of the places before returning to his starting place.

(i) Show that, using the nearest neighbour algorithm starting from Sagrada Familia (S), the total travelling time for Salvadore is 145 minutes. *(3 marks)*

(ii) Explain why your answer to part (c)(i) is an upper bound for the minimum travelling time for Salvadore. *(2 marks)*

(iii) Salvadore starts from Sagrada Familia (S) and then visits Ramblas (R). Given that he visits Nou Camp (N) before Park Guell (P), find an improved upper bound for the total travelling time for Salvadore. *(3 marks)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

(a) Gill is solving a travelling salesperson problem.

(i) She finds the following upper bounds: 7.5, 8, 7, 7.5, 8.5 .

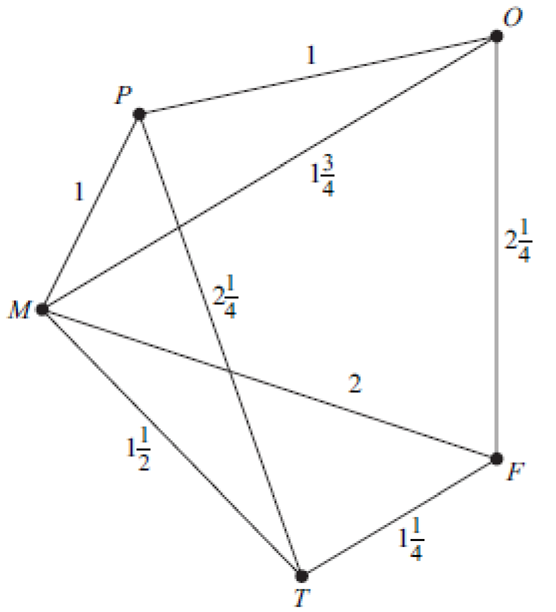
Write down the best upper bound. *(1 mark)*

(ii) She finds the following lower bounds: 6.5, 7, 6.5, 5, 7.

Write down the best lower bound. *(1 mark)*

(b) George is travelling by plane to a number of cities. He is to start at F and visit each of the other cities at least once before returning to F.

The diagram shows the times of flights, in hours, between cities. Where no time is shown, there is no direct flight available.



(i) Complete **Figure 2** to show the minimum times to travel between all pairs of cities. *(2 marks)*

(ii) Find an upper bound for the minimum total flying time by using the route *FTPOMF*. *(1 mark)*

(iii) Using the nearest neighbour algorithm starting from F, find an upper bound for the minimum total flying time. *(4 marks)*

(iv) By deleting F, find a lower bound for the minimum total flying time. *(5 marks)*

3 Mark is driving around the one-way system in Leicester. The following table shows the times, in minutes, for Mark to drive between four places: A, B, C and D. Mark decides to start from A, drive to the other three places and then return to A.

Mark wants to keep his driving time to a minimum.

From \ To	***A***	***B***	***C***	***D***
A	–	8	6	11
B	14	–	13	25
C	14	9	–	17
D	26	10	18	–

(a) Find the length of the tour $ABCDA$. *(2 marks)*

(b) Find the length of the tour $ADCBA$. *(1 mark)*

(c) Find the length of the tour using the nearest neighbour algorithm starting from A. *(4 marks)*

(d) Write down which of your answers to parts (a), (b) and (c) gives the best upper bound for Mark's driving time. *(1 mark)*

6 (a) Mark is staying at the Grand Hotel (G) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge (A), the National Theatre (N), Parliament House (P) and the Royal Palace (R).

The figures in the table represent the walking times, in seconds, between the places.

	Grand Hotel (G)	**Aker Brygge (A)**	**National Theatre (N)**	**Parliament House (P)**	**Royal Palace (R)**
Grand Hotel (G)	–	165	185	65	160
Aker Brygge (A)	165	–	155	115	275
National Theatre (N)	185	155	–	205	125
Parliament House (P)	65	115	205	–	225
Royal Palace (R)	160	275	125	225	–

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.

(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour. *(4 marks)*

(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour. *(5 marks)*

(iii) The walking time for an optimal tour is T seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about T. *(1 mark)*

(b) Mark then intends to start from the Grand Hotel (G), visit three museums, Ibsen (I), Munch (M) and Viking (V), and return to the Grand Hotel. He uses public transport. The table shows the minimum travelling times, in minutes, between the places.

From \ To	**Grand Hotel (G)**	**Ibsen (I)**	**Munch (M)**	**Viking (V)**
Grand Hotel (G)	–	20	17	30
Ibsen (I)	15	–	32	16
Munch (M)	26	18	–	21
Viking (V)	19	27	24	–

(i) Find the length of the tour $GIMVG$. *(1 mark)*

(ii) Find the length of the tour $GVMIG$. *(1 mark)*

(iii) Find the number of different possible tours for Mark. *(1 mark)*

(iv) Write down the number of different possible tours for Mark if he were to visit n museums, starting and finishing at the Grand Hotel. *(1 mark)*

	M	P	O	T	F
M	–	1	$1\frac{3}{4}$	$1\frac{1}{2}$	2
P	1	–	1	$2\frac{1}{4}$	
O	$1\frac{3}{4}$	1	–		$2\frac{1}{4}$
T	$1\frac{1}{2}$	$2\frac{1}{4}$		–	$1\frac{1}{4}$
F	2		$2\frac{1}{4}$	$1\frac{1}{4}$	–