

Roots of Quadratics

This...

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Leads to...

$$ax^2 + bx + c = 0 \quad \Leftrightarrow \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Some common related roots

New Roots	New Sum	New Product
α^2, β^2	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	$\alpha^2\beta^2 = (\alpha\beta)^2$
$\frac{1}{\alpha}, \frac{1}{\beta}$	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$	$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta}$
α^3, β^3	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	$(\alpha\beta)^3$
$5\alpha + \frac{1}{\alpha}, 5\beta + \frac{1}{\beta}$	$5\alpha + \frac{1}{\alpha} + 5\beta + \frac{1}{\beta}$ $= 5(\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$	$\left(5\alpha + \frac{1}{\alpha}\right)\left(5\beta + \frac{1}{\beta}\right)$ $= 25\alpha\beta + \frac{5(\alpha + \beta)^2}{\alpha\beta} - 10 + \frac{1}{\alpha\beta}$

For a cubic with roots $\alpha, \beta, \gamma \dots$

This...

$$(x - \alpha)(x - \beta)(x - \gamma) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma = 0$$

Leads to...

$$ax^3 + bx^2 + cx + d = 0 \quad \Leftrightarrow \quad x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$$

$$\text{Sum of roots} = \alpha + \beta + \gamma = -\frac{b}{a}$$

$$\text{Sum of pairs} = \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{Product of roots} = \alpha\beta\gamma = -\frac{d}{a}$$

What about a quartic, quintic etc?

How to get from...

$$\alpha + \beta \quad \text{and} \quad \alpha\beta$$

To...

1)

$$\alpha^2 + \beta^2 \quad \text{and} \quad \alpha^2\beta^2$$

2)

$$\alpha^3 + \beta^3 \quad \text{and} \quad \alpha^3\beta^3$$

3)

$$\frac{1}{\alpha} + \frac{1}{\beta} \quad \text{and} \quad \frac{1}{\alpha} \times \frac{1}{\beta}$$

4)

$$5\alpha + \frac{1}{\alpha} + 5\beta + \frac{1}{\beta} \quad \text{and} \quad \left(5\alpha + \frac{1}{\alpha}\right)\left(5\beta + \frac{1}{\beta}\right)$$