**A Level Questions of a Problem-Solving Variety**

1. Prove from first principles that
   1. If , then
   2. If , then
2. A colony of ants grows exponentially according to the formula

Where is the number of ants in the colony (measured in thousands), is time measured in days and are both constants. Initially, the colony of ants numbers 1000 and after 10 days the population has risen to 1500.

1. Compute the population of ants in the colony after 15 days.

In a different colony of ants, a graph of against is plotted resulting in a straight line. The gradient of the line drawn is 0.012 and it is known that the initial population of ants is equal to that in the previous population.

1. Find the difference between the number of ants in both colonies after 10 days.
2. Calculate the total number of days required for this colony of ants to first exceed the number of ants after 10 days in the first colony.
3. Equivalent expressions when using logarithms are . By considering a logarithm of base (,
   1. Show that .

Hence, find the exact value of

* 1. .

And show that

* 1. .
  2. Without using a calculator, explain why

1. The position of a particle is modelled by the parametric equations

and

Where . The particle traces out a trajectory, , from to .

1. Show that the trajectory satisfies the equation
2. By considering the domain and range of , sketch the curve , clearly indicating the minimum point.
3. Find an expression for the gradient of the trajectory and deduce that for , the gradient function is a decreasing function. (You do not need to be concerned about the situation when ).
4. Explain why the choice of is valid in the Binomial Expansion of , where

Hence, by computing the first three terms of the binomial expansion of , find an approximation for

1. Without using differentiation, find the maximum and minimum values of , where

and find the first value of for which a maximum and a minimum occur, correct to one decimal place.

* 1. By first finding the partial fractions within the integral, find, giving your answer as a natural logarithm
  2. Find the value of

Giving your answer in an exact form.

1. Prove the Newton-Raphson formula for finding an approximate solution to the equation .
2. In this question, you may assume that there is only one root of . Using the Newton-Raphson formula, find an approximation to the solution of .
3. Money is invested in a bank account that pays 1.2% interest per annum. Initially, I invest £1,000. The total amount in the account after years is given by , where is a geometric sequence.
   1. Show that
   2. Find how many years I will have to wait for the amount in the account to first exceed £1,300.
4. By first writing the expression in the form of an exponential with base show that
   1. Prove the result in a. again, but this time taking natural logarithms first.
   2. Hence, find an expression for the exact value of for which the curve takes has a gradient of
5. Let and
   1. Explain why
      1. for all values of .
      2. as
   2. How are your answers to a. affected if instead, ?
6. The vector  and . The vector has the same magnitude and is parallel to the vector , whilst the position vector of the point is parallel to and .  
     
   Find the vector and .
7. A conical tank of radius m and height m is initially full. At time the tank begins to link from three holes: the first leaks at a rate of 10% the current volume, the second at a rate of 20% the current volume and the third at a rate of 30% the current volume. To stop the tank from running empty, liquid is added at a rate of 1m3/s.
   1. Explain why, for all times, .
   2. As time progresses, the level of water in the tank settles to a volume , where the input of 1m3/s is balanced by the loss of liquid from the tank. Find the value of
      1. Without solving the differential equation at all.
      2. By first solving the differential equation.
   3. Show further that

and **by solving the differential equation above**, find the radius of the liquid in the tank and hence the height of liquid in the tank as the volume approaches its limiting value.

* 1. Can you find the same result by not solving the differential equation above?

1. Baby weight, is conjected to increase logarithmically with time, . The population of baby weights are grouped into percentiles, with a baby on the 91st percentile for their weight meaning that they are among the top 9% of baby weights nationally. Immediately after a birth, the weight of a baby fluctuates and settles down usually after a couple of weeks.

For a baby tracking the 91st percentile after 2 weeks, they would weigh 4.5kg and after 52 weeks they would weigh 11.1kg. After 6 months, a baby following this weight gain line would be expected to weigh around 9.2kg.

To test the conjecture, a model of the form

Is proposed.

1. Why could such a model be proposed when a ‘logarithmic increase’ is stated?
2. Comment on the accuracy of the model by comparing the model at 6 months against the real-life data.
3. A logistic equation is defined by the rate of change of being proportional to the product of with . In this logistic equation, for all time, and assume that is a continuous variable.

Suppose that at , and that at time , , where is a constant. Assume further that the model is valid only for .

Show that the constant of proportionality is

and deduce in the case of that for the specified domain.

1. Recall that for small values of , . By considering the first three non-zero terms of a binomial expansion and picking a suitable value of , find an approximate value for , correct to three decimal places. Comment on the accuracy of your result by comparing the approximate value against the true value.
2. Find the greatest possible domain of the function . For the domain , find the range of the function .
3. Let

and

Where , be parametric equations of the curve .

* 1. Give a reason why the range of the function is .
  2. Show that the domain of is .
  3. Find the values of for which is an increasing function. Find also the values of for which is a decreasing function.
  4. Using parts a. to c., sketch the curve
  5. Show that and hence deduce a form for the curve . What type of function is

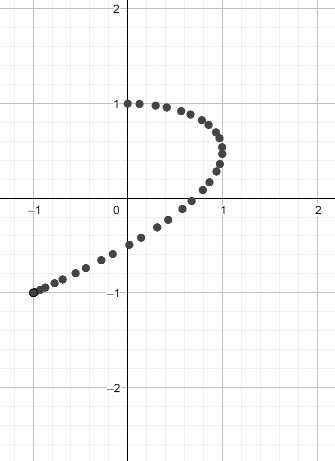
1. Let and. By considering the partial fractions of, find an expression for the integral   
   (**Hint**: you will, at some point, need to use the fact that ; see the formula book)
2. The radiation received on Earth from a Pulsar has Intensity and for is modelled by

Where is a constant.

1. State the values of the maximum and minimum intensity of the radiation received on Earth and find, in terms of , the times at which the first two maxima and minima occur.
2. Given also that when and when and that exactly one maximum and minimum occur during the time interval , explain why .
3. The points , and are the coordinates of the vertices of the triangle in three dimensions.  
   Find both the perimeter and area of the triangle .
4. The position of a particle is modelled by the parametric equations

and

Where is time in seconds and The trajectory of the curve is given by , an expression of the form . The diagram below shows the position of the particle for various times throughout the specified domain.



Find the gradient of the curve at the times when is zero and comment on their value. Is this implied by the graph of above?

Find an expression for the trajectory of the curve for those times which the trajectory is a  
decreasing function.

1. Let , and , and .
   1. Solve
   2. Consider the equation
      1. Show that is a root.
      2. Show that a non-zero root, , to the equation exists between and 1.
      3. Show that a Newton-Raphson formula to find an approximate value of the root is given by

and hence find, using a suitable first approximation to the root, the value of correct to three significant figures.

1. The parabola defined by passes through the points , and . The point lies on the curve and has coordinates . Find the exact value of
2. A parabola is symmetric about the line and has a repeated root. The parabola also passes through the point Find the exact value of the -intercept of the parabola.
3. A circle has centre and passes through the points . The tangent to the circle at the point is parallel to the tangent at the point . Find the equation of the tangent at the point .
4. The function, where has domain Prove that the range of is and hence find two possible domains in for which is a one-to-one function.
5. Let . Find the range of . Find the domain of the inverse function, if the domain is restricted to .
6. Prove that there are infinitely many prime numbers.
7. Prove that is irrational.
8. Prove that is irrational.
9. Let Show that
10. A cylinder of height is to be produced so that when 2m3/s of liquid is added to the cylinder and 80% of the volume is lost at that given instant in time, the cylinder remains perfectly full. Assume that the cylinder is initially empty. Show that and hence find the radius, correct to one decimal place, of the cylinder.  
    1. Find the value of
    2. Find the value of

And by picking suitable values of and , show that your answer to part a. matches with the answer in part b.

1. Let and for . Using parametric integration, find the value of

For the values of for which and are defined.

1. Let

and suppose that the curve satisfies the conditions that when . Find the exact value of when

1. Let , where and are constants. Show that

And hence deduce that

1. The function has exactly two stationary points, and let and be the - coordinates of those stationary points. Let . Show that , that and by using Newton-Raphson iteration, find the values of and , correct to two decimal places.