Lots of Hypothesis Testing with the Binomial Expansion

One Tailed

1) Lindsay Lohan complains that a dice she is using is biased so that she is less likely to get a 6. She decides to test at the 5% significance level the hypothesis that the dice is biased against a six. Lindsay throws the dice 20 times and gets just one six.

   a) Write down the null and alternative hypothesis that Lindsay will use.
   b) Find the p-value for this test.
   c) What is Lindsay's conclusion?

2) Sir David Attenborough complains that a dice he is using is biased so that he is more likely to get a 1 than any other number. He decides to test at the 10% significance level the hypothesis that the dice is biased towards a 1. Sir David Attenborough throws the dice 12 times and gets 5 ones.

   a) Write down the null and alternative hypothesis that Sir David Attenborough will use.
   b) Find the p-value for this test.
   c) What is Sir David Attenborough's conclusion?

3) A train company claims that a particular service is on time in 90% of journeys. Francis Bourgeois thinks it is less than this. He decides to test this at the 5% significance level. In his next 15 journeys, the train is on time 10 times.

   a) Write down the null and alternative hypothesis that Francis should use.
   b) Find the p-value for this test.
   c) What is Francis's conclusion?

4) David Tennant believes that his Tardis is accurate in 80% of journeys. The BBC claims that it has made improvements to the Tardis and that the Tardis is now accurate more often. David decides to test this at the 10% significance level. In his next 16 adventures, the Tardis is on time 15 times.

   a) Write down the null and alternative hypotheses that David should use.
   b) Find the p-value for this hypothesis test.
   c) What is David's conclusion?

Two Tailed

1) It is known from past records that 1 in 5 singles produced by Ariana Grande are pretty awesome. To monitor production, a random sample of 25 singles was taken and the number of such singles that are pretty awesome was, sadly on this occasion, found to be only 1. Using a 5% level of significance, conduct a 2-tail test of the hypothesis that 1 in 5 singles are pretty awesome.

2) The probability that a student dislikes learning about hypothesis testing is 60%. After doing some amazing teaching about hypothesis testing, 7 of 20 students in Mr Colman's class say they dislike learning about hypothesis testing. Conduct a hypothesis test at the 5% level of significance to determine whether Mr Colman's teaching has changed the probability that a student dislikes learning about hypothesis testing. What about if we tested at 1%, one tailed instead?

3) In Norwich, the proportion of inhabitants from the suburb of Sprowston is known to be 0.3. A sample of 12 employees of Aviva (a large company based in the city) is obtained and it is found that 2 of them are from Sprowston. Carry out a test at the 5% significance level to determine whether the proportion of employees at Aviva from Sprowston is similar to the proportion of the city as a whole.

4) A television program called 'Bake it Off' changed three of its presenters when it moved TV channel and the other three quit. The program was viewed by 20% of a population of 50 million. The producers want to determine if this change has affected viewing figures. From a random sample of 100 from the population, 28 said they watched the program. Carry out a suitable hypothesis test at the 5% significance level to determine whether there is sufficient evidence to suggest viewing figures have changed.

You Decide

1) Brad Pitt planted 25 seeds in his greenhouse. He has read in a gardening book that the probability of one of these seeds germinating is 0.25. **Ten** of Brad's seeds germinated. He claims that the gardening book has underestimated this probability. Test Brad's claim at the 5% significance level.

2) Angelina Jolie regularly takes a cheap taxi to the acting studios five times a week. Over a long period of time, she finds that the taxi is late once a week, on average. The taxi firm changes her driver and in the first week with the new driver, the taxi is late **three** times. Angelina thinks that the rate of taxi lateness is different to before. Test Angelina's claim at the 5% significance level.

3) Peter Andre has a six-sided dice which he suspects is biased so that it is more likely to show a six than if it were fair. Peter throws the dice 30 times and it shows a six on 9 throws. Test at the 5% significance level whether Peter Andre's suspicion is justified.

4) It is known that under the standard treatment for calming down overexcited fans at Peter Andre concerts, 9.7% of overexcited fans who use the treatment experience side effects. In a trial of a new treatment, 450 overexcited fans were selected and the number, X, that experienced side effects was noted. It was found that 51 of the 450 overexcited fans who use the treatment experienced side effects. Test, at the 10% significance level, whether the proportion of overexcited fans experiencing side effects is greater under the new treatment than under the standard treatment.

5) 55% of the pupils in a large school are Peter Andre fans. A student claims that the probability that a Peter Andre fan, rather than a someone with a more sensible taste in music and minor celebrities, becomes Head Student is greater than 0.55. As evidence for his claim he says that 6 of the last 8 Head Students have been Peter Andre fans. Test this claim at the 10% significance level.

6) Research has shown that drug A is effective in treating 32% of people with an irrational fascination of Peter Andre. In a trial, drug B is given to a random sample of 1000 people with an irrational fascination of Peter Andre, and it is found that the drug is effective in treating 290 of these people. Test at the 2.5% significance level whether there is evidence that drug B is effective in a different proportion of people to drug A.

Finding the values that fall within the critical region

1) Elon Musk has a rocket company called SpaceX in which he re-use the three rocket boosters used on each launch by landing them back on the pad after launch. His rocket boosters are rated for a one in six chance of landing back on the pad, but Elon is aiming for a greater success rate. Elon launches 30 rockets and all three boosters land back on the pad successfully on 9 occasions. He considers performing a hypothesis test with a 5% significance level.

   a) What values form the critical region for the hypothesis test?
   b) Does the observed value lie within the critical region?
   c) Given this information, should you accept or reject H_0 in this scenario?
   d) Conclude in context.

2) It is known that under normal market conditions, the probability of someone purchasing a particular book about *the mighty Saturn V rocket* from Amazon is 9.7%. In a trial of a new marketing campaign, 450 page views were monitored by Jeff Bezos and it was found that 51 of them led to a sale of the book about *the mighty Saturn V rocket*. Jeff considers performing a hypothesis test with a 10% level of significance to investigate whether the probability of someone purchasing this book is greater than 9.7%.

   a) What values form the critical region for the hypothesis test?
   b) Does the observed value lie within the critical region?
   c) Given this information, should you accept or reject H_0 in this scenario?
   d) Conclude in context.

3) It is reckoned that 55% of celebrities claim to be interested in purchasing a ride on Richard Branson's Virgin Galactic space flight experience. Richard claims that the probability that a celebrity is interested in taking a spaceflight with Virgin Galactic is greater than 0.55. As evidence for this claim, he says that six of the last eight celebrities have said that they are interested. Richard Branson considers performing a hypothesis test with a 10% level of significance.

   a) What values form the critical region for the hypothesis test?
   b) Does the observed value lie within the critical region?
   c) Given this information, should you accept or reject H_0 in this scenario?
   d) Conclude in context.

4) Tim Peake was an astronaut with the European Space Agency. For each astronaut test he took, he succeeded in half of them. After writing a new book on *The Astronaut Selection Test*, and thereby revising his own knowledge, Tim took ten tests

and succeeded in seven of them. He considers performing a hypothesis test with a 10% significance level to see if his likelihood of passing astronaut tests has improved.

a) What values form the critical region for the hypothesis test?
b) Does the observed value lie within the critical region?
c) Given this information, should you accept or reject H_0 in this scenario?
d) Conclude in context.

5) Artemis II Mission Specialist Christina Koch is good at identifying lunar craters, but estimates that she is successful only 70% of the time. Whilst on shift three of lunar observations, with Commander Reid Wiseman in support, Christina correctly identified nine of the first ten lunar craters that she observed. Reid thinks Christina is better than she says and, whilst eating a tortilla with Nutella on it, considers performing a hypothesis test at a 5% significance level on Christina's first ten lunar crater identifications.

a) What values form the critical region for the hypothesis test?
b) Does the observed value lie within the critical region?
c) Given this information, should you accept or reject H_0 in this scenario?
d) Conclude in context.

Lots of Hypothesis Testing with the Binomial Expansion

One Tailed

1) Lindsay Lohan complains that a dice she is using is biased so that she is less likely to get a 6. She decides to test at the 5% significance level the hypothesis that the dice is biased against a six. Lindsay throws the dice 20 times and gets just one six.

   a) Write down the null and alternative hypothesis that Lindsay will use.
   b) Find the p-value for this test. The p-value is 0.130
   c) What is Lindsay's conclusion? 0.130 > 0.05 so do not reject H_0. There is insufficient evidence to suggest that the dice is biased.

2) Sir David Attenborough complains that a dice he is using is biased so that he is more likely to get a 1 than any other number. He decides to test at the 10% significance level the hypothesis that the dice is biased towards a 1. Sir David Attenborough throws the dice 12 times and gets 5 ones.

   a) Write down the null & alternative hypothesis that Sir David Attenborough will use.
   b) Find the p-value for this test. The p-value is 0.0364
   c) What is Sir David Attenborough's conclusion? 0.0364 < 0.1 so reject H_0. The evidence suggests that the dice is biased towards a 1.

3) A train company claims that a particular service is on time in 90% of journeys. Francis Bourgeois thinks it is less than this. He decides to test this at the 5% significance level. In his next 15 journeys, the train is on time 10 times.

   a) Write down the null and alternative hypothesis that Francis should use.
   b) Find the p-value for this test. The p-value is 0.012
   c) What is Francis's conclusion? 0.012 < 0.05 so reject H_0. The evidence suggests that the train is on time in less than 90% of journeys.

4) David Tennant believes that his Tardis is accurate in 80% of journeys. The BBC claims that it has made improvements to the Tardis and that the Tardis is now accurate more often. David decides to test this at the 10% significance level. In his next 16 adventures, the Tardis is on time 15 times.

   a) Write down the null and alternative hypotheses that David should use.
   b) Find the p-value for this hypothesis test. The p-value is 0.141
   c) What is David's conclusion? 0.141 > 0.1, so do not reject H_0. There is insufficient evidence that the Tardis is accurate more than 80% of the time.

Two Tailed

1) It is known from past records that 1 in 5 singles produced by Ariana Grande are pretty awesome. To monitor production, a random sample of 25 singles was taken and the number of such singles that are pretty awesome was, sadly on this occasion, found to be only 1. Using a 5% level of significance, conduct a 2-tail test of the hypothesis that 1 in 5 singles are pretty awesome.

2) The probability that a student dislikes learning about hypothesis testing is 60%. After doing some amazing teaching about hypothesis testing, 7 of 20 students in Mr Colman's class say they dislike learning about hypothesis testing. Conduct a hypothesis test at the 5% level of significance to determine whether Mr Colman's teaching has changed the probability that a student dislikes learning about hypothesis testing. What about if we tested at 1%, one tailed instead?

3) In Norwich, the proportion of inhabitants from the suburb of Sprowston is known to be 0.3. A sample of 12 employees of Aviva (a large company based in the city) is obtained and it is found that 2 of them are from Sprowston. Carry out a test at the 5% significance level to determine whether the proportion of employees at Aviva from Sprowston is similar to the proportion of the city as a whole. Two tailed. $P(x \leq 2) = 0.0253.\ 0.0253 > 0.025$ so accept H_0.

4) A television program called 'Bake it Off' changed three of its presenters when it moved TV channel and the other three quit. The program was viewed by 20% of a population of 50 million. The producers want to determine if this change has affected viewing figures. From a random sample of 100 from the population, 28 said they watched the program. Carry out a suitable hypothesis test at the 5% significance level to determine whether there is sufficient evidence to suggest viewing figures have changed. Two tailed. $P(x \geq 28) = 0.0342.\ 0.0342 < 0.025$ so reject H_0.

You Decide

1) Brad Pitt planted 25 seeds in his greenhouse. He has read in a gardening book that the probability of one of these seeds germinating is 0.25. **Ten** of Brad's seeds germinated. He claims that the gardening book has underestimated this probability. Test Brad's claim at the 5% significance level. One tailed. H_0:p=0.25, H_1:p>0.25, P(x≥10)=0.071 so accept H_0. Critical region is x≥11. 10 is outside CR, so accept H_0.

2) Angelina Jolie regularly takes a cheap taxi to the acting studios five times a week. Over a long period of time, she finds that the taxi is late once a week, on average. The taxi firm changes her driver and in the first week with the new driver, the taxi is late **three** times. Angelina thinks that the rate of taxi lateness is different to before. Test Angelina's claim at the 5% significance level. Two tailed. H_0:p=0.2, H_1:p≠0.2, P(x≥3)=0.05792 so accept H_0. Critical region is x≥4. 3 is outside CR, so accept H_0.

3) Peter Andre has a six-sided dice which he suspects is biased so that it is more likely to show a six than if it were fair. Peter throws the dice 30 times and it shows a six on 9 throws. Test at the 5% significance level whether Peter Andre's suspicion is justified. One tailed. H_0:p=1/6, H_1:p>1/6, P(x≥9)=0.05057 so accept H_0. Or critical region is…

4) It is known that under the standard treatment for calming down overexcited fans at Peter Andre concerts, 9.7% of overexcited fans who use the treatment experience side effects. In a trial of a new treatment, 450 overexcited fans were selected and the number, X, that experienced side effects was noted. It was found that 51 of the 450 overexcited fans who use the treatment experienced side effects. Test, at the 10% significance level, whether the proportion of overexcited fans experiencing side effects is greater under the new treatment than under the standard treatment. One tailed. $P(x \geq 51) = 0.138$. $0.138 > 0.10$ so accept H_0.

5) 55% of the pupils in a large school are Peter Andre fans. A student claims that the probability that a Peter Andre fan, rather than a someone with a more sensible taste in music and minor celebrities, becomes Head Student is greater than 0.55. As evidence for his claim he says that 6 of the last 8 Head Students have been Peter Andre fans. Test this claim at the 10% significance level. One tailed.

6) Research has shown that drug A is effective in treating 32% of people with an irrational fascination of Peter Andre. In a trial, drug B is given to a random sample of 1000 people with an irrational fascination of Peter Andre, and it is found that the drug is effective in treating 290 of these people. Test at the 2.5% significance level whether there is evidence that drug B is effective in a lower proportion of people than drug A. Two tailed. $P(x \leq 290) = 0.022$. $0.022 > 0.115$ so accept H_0.

Finding the values that fall within the critical region

1) Elon Musk has a rocket company called SpaceX in which he re-use the three rocket boosters used on each launch by landing them back on the pad after launch. His rocket boosters are rated for a one in six chance of landing back on the pad, but Elon is aiming for a greater success rate. Elon launches 30 rockets and all three boosters land back on the pad successfully on 9 occasions. He considers performing a hypothesis test with a 5% significance level.

   a) What values form the critical region for the hypothesis test? 10 - 30
   b) Does the observed value lie within the critical region? No
   c) Given this information, should you accept or reject H_0 in this scenario? Accept H_0
   d) Conclude in context.

2) It is known that under normal market conditions, the probability of someone purchasing a particular book about *the mighty Saturn V rocket* from Amazon is 9.7%. In a trial of a new marketing campaign, 450 page views were monitored by Jeff Bezos and it was found that 51 of them led to a sale of the book about *the mighty Saturn V rocket*. Jeff considers performing a hypothesis test with a 10% level of significance to investigate whether the probability of someone purchasing this book is greater than 9.7%.

   a) What values form the critical region for the hypothesis test? 53 - 450
   b) Does the observed value lie within the critical region? No
   c) Given this information, should you accept or reject H_0 in this scenario? Accept H_0
   d) Conclude in context.

3) It is reckoned that 55% of celebrities claim to be interested in purchasing a ride on Richard Branson's Virgin Galactic space flight experience. Richard claims that the probability that a celebrity is interested in taking a spaceflight with Virgin Galactic is greater than 0.55. As evidence for this claim, he says that six of the last eight celebrities have said that they are interested. Richard Branson considers performing a hypothesis test with a 10% level of significance.

   a) What values form the critical region for the hypothesis test? 7 - 8
   b) Does the observed value lie within the critical region? No
   c) Given this information, should you accept or reject H_0 in this scenario? Accept H_0
   d) Conclude in context.

4) Tim Peake was an astronaut with the European Space Agency. For each astronaut test he took, he succeeded in half of them. After writing a new book on *The Astronaut Selection Test*, and thereby revising his own knowledge, Tim took ten tests

and succeeded in seven of them. He considers performing a hypothesis test with a 10% significance level to see if his likelihood of passing astronaut tests has improved.

a) What values form the critical region for the hypothesis test? 8 - 10
b) Does the observed value lie within the critical region? No
c) Given this information, should you accept or reject H_0 in this scenario? Accept H_0
d) Conclude in context.

5) Artemis II Mission Specialist Christina Koch is good at identifying lunar craters, but estimates that she is successful only 70% of the time. Whilst on shift three of lunar observations, with Commander Reid Wiseman in support, Christina correctly identified nine of the first ten lunar craters that she observed. Reid thinks Christina is better than she says and, whilst eating a tortilla with Nutella on it, considers performing a hypothesis test at a 5% significance level on Christina's first ten lunar crater identifications.

a) What values form the critical region for the hypothesis test? 10
b) Does the observed value lie within the critical region? Yes
c) Given this information, should you accept or reject H_0 in this scenario? Reject H_0
d) Conclude in context.