Harmonic Form

$$a\sin x \pm b\cos x = R\sin(x \pm \alpha)$$
$$a\cos x \pm b\sin x = R\cos(x \mp \alpha)$$

Where

$$R = \sqrt{a^2 + b^2}$$

$R\cos\alpha = a$ and $R\sin\alpha = b$

$$\Rightarrow \cos\alpha = \frac{a}{R},\ \sin\alpha = \frac{b}{R},\ \tan\alpha = \frac{b}{a}$$

Harmonic Form Teaching Activity

1. Draw $y = 3\sin x + 2\cos x$. Make observations.
2. Experiment with other versions of $y = a\sin x + b\cos x$ making further observations.

3. Realize that $y = 3\sin x + 2\cos x$ can be written in the form $y = R\sin(x+\alpha)$ and try to suggest reasons for these values ($R \approx 3.6,\ \alpha \approx 0.58$).

4. Do the algebra. Equating both expressions and using double angle formulae:

$$3\sin x + 2\cos x = R\sin(x+\alpha)$$
$$3\boldsymbol{sinx} + 2\boldsymbol{cosx} = R\boldsymbol{sinx}cos\alpha + R\boldsymbol{cosx}sin\alpha$$

   Therefore:

$$3\boldsymbol{sinx} = R\boldsymbol{sinx}cos\alpha \Rightarrow 3 = Rcos\alpha \Rightarrow cos\alpha = \frac{3}{R}$$

$$2\boldsymbol{cosx} = R\boldsymbol{cosx}sin\alpha \Rightarrow 2 = Rsin\alpha \Rightarrow sin\alpha = \frac{2}{R}$$

$$R = \sqrt{3^2 + 2^2} \qquad \text{and} \qquad \tan\alpha = \frac{2}{3}$$

5. Textbook or exam questions where students convert equations into harmonic form.
6. This question…

   Rewrite $\sqrt{3}cosx - sinx$ in the form

   a) $Rcos(x+\alpha)$
   b) $Rsin(x-\alpha)$
   c) Prove via graph transformations that your answers to part (a) and (b) are the same.