

Useful Vector Formulae for Further Maths

Angle between two vectors	$\cos\theta = \frac{a \cdot b}{ a b }, \quad \sin\theta = \frac{a \times b}{ a b }$
Cross product of two vectors = area of parallelogram formed by the vectors	$a \times b = a b \sin\theta\hat{n}$
Volume of parallelepiped (aka the scalar triple product)	$a \cdot b \times c$
Volume of tetrahedron	$\frac{1}{6}(a \cdot b \times c)$
Some properties of the cross product	$a \times b = -(b \times a)$ $ma \times nb = mn(a \times b)$ $a \times (b + c) = a \times b + a \times c$
Vector equation of line takes the form	$r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ $r = a + \lambda d$ $\Leftrightarrow r - a = \lambda d$ $\Leftrightarrow (r - a) \times d = 0$ $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6}$
Vector equation of plane takes the form	$r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ $r \cdot n = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot n$ $n_1x + n_2y + n_3z + d = 0$

$$a \times b = 0 \Leftrightarrow \text{Vectors parallel}$$

$$a \cdot b = 0 \Leftrightarrow \text{Vectors perpendicular}$$

$$a \cdot (a \times b) = 0 \Leftrightarrow a, b, c \text{ coplanar (because vol parallelepiped} = 0)$$