

Chain, Product, Quotient Rule Recap

Differentiate these.

Chain Rule

$y = (3x + 4)^3$	$y = \sin(2x)$	$y = e^{2x}$
$y = (3x^3 + 1)^3$	$y = \sin(x^2)$	$y = e^{(x^3)}$
$y = \frac{4}{4 - x^4}$	$y = \frac{1}{(\operatorname{cosec} x)}$	$y = \ln\left(\frac{1}{e^x}\right)$
$y = \sqrt{\sqrt{x} + 1}$	$y = \sin(\cos(\tan x))$	$y = e^{(e^x)}$

Product Rule

$y = (3x + 4)(2x - 3)$	$y = x^2 \sin x$	$y = xe^x$
$y = (3x^2 + 4)(2x^4 + 3)$	$y = \sin x \cos x$	$y = x^3 e^x$
$y = \sqrt{3x} \sqrt{2x}$	$y = \sin x \cos x \tan x$	$y = x \ln x$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$	$y = \sin(x^2) \cos(x^2)$	$y = e^{x^2} \ln(x^2)$

Quotient Rule

$y = \frac{x + 1}{2x + 1}$	$y = \frac{3x^2}{\sin x}$	$y = \frac{2x}{e^x}$
$y = \frac{x^2 + 1}{x^2 - 1}$	$y = \tan x$	$y = \frac{e^x}{e^{-x}}$
$y = \frac{x^2}{\sqrt{x}}$	$y = \frac{x^2}{\tan x}$	$y = \frac{e^{-4x}}{4e^{4x}}$
$y = \sqrt[3]{\frac{x + 1}{x - 1}}$	$y = \frac{\cot x}{2 \sec x}$	$y = \frac{\ln(x^2)}{e^{x^2}}$

Chain, Product, Quotient Rule Recap - Answers

Differentiate these.

Chain Rule

$y = (3x + 4)^3$ $\frac{dy}{dx} = 9(3x + 4)^2$	$y = \sin(2x)$ $\frac{dy}{dx} = 2\cos(2x)$	$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$
$y = (3x^3 + 1)^3$ $\frac{dy}{dx} = 3(3x^3 + 1)^2(9x^2)$ $= 27x^2(3x^3 + 1)^2$	$y = \sin(x^2)$ $\frac{dy}{dx} = 2x\cos(x^2)$	$y = e^{(x^3)}$ $\frac{dy}{dx} = 3x^2e^{(x^3)}$
$y = \frac{4}{4 - x^4} = 4(4 - x^4)^{-1}$ $\frac{dy}{dx} = -4(4 - x^4)^{-2} \times -4x^3$ $= \frac{16x^3}{(4 - x^4)^2}$	$y = \frac{1}{\left(\frac{1}{\operatorname{cosec}x}\right)} = \frac{1}{\sin x}$ $= (\sin x)^{-1}$ $\frac{dy}{dx} = -(\sin x)^{-2} \times \cos x$ $= \frac{-\cos x}{(\sin x)^2} = \operatorname{cot}x \operatorname{cosec}x$	$y = \ln\left(\frac{1}{e^x}\right)$ $\frac{dy}{dx} = \frac{1}{\frac{1}{e^x}} \times -e^{-x}$ $= -e^x \times e^{-x}$ $= -e^0$ $= -1$ <p style="text-align: right; font-size: small;">(convince yourself that this is the case)</p>
$y = \sqrt{\sqrt{x} + 1} = \left(x^{\frac{1}{2}} + 1\right)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{1}{2}} + 1\right)^{-\frac{1}{2}} \times \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{1}{4} \times \frac{1}{\sqrt{\sqrt{x} + 1}} \times \frac{1}{\sqrt{x}}$ $= \frac{1}{4\sqrt{x}\sqrt{\sqrt{x} + 1}}$	$y = \sin(\cos(\tan x))$ $\frac{dy}{dx} = -\cos(\cos(\tan x))\sin(\tan x)\sec^2 x$	$y = e^{e^x}$ $\frac{dy}{dx} = e^{e^x}e^x = e^{e^x+x}$

Product Rule

$y = (3x + 4)(2x - 3)$ $\frac{dy}{dx} = 2(3x + 4) + 3(2x - 3)$ $= 12x - 1$	$y = x^2 \sin x$ $\frac{dy}{dx} = x^2 \cos x + 2x \sin x$	$y = x e^x$ $\frac{dy}{dx} = x e^x + e^x$ $= (x + 1) e^x$
$y = (3x^2 + 4)(2x^4 + 3)$ $\frac{dy}{dx} = 8x^3(3x^2 + 4) + 6x(2x^4 + 3)$ $= 2x[4x^2(3x^2 + 4) + 3(2x^4 + 3)]$ $= 2x[12x^4 + 16x^2 + 6x^4 + 9]$ $= 2x[18x^4 + 16x^2 + 9]$	$y = \sin x \cos x$ $\frac{dy}{dx} = \cos^2 x - \sin^2 x$	$y = x^3 e^x$ $\frac{dy}{dx} = 3x^2 e^x + x^3 e^x$ $= x^2 e^x (x + 3)$
$y = \sqrt{3x} \sqrt{2x}$ $= (6x^2)^{\frac{1}{2}} = \sqrt{6x}$ $\frac{dy}{dx} = \sqrt{6}$ <p>(easy method?)</p>	$y = \sin x \cos x \tan x$ $= \sin x \cos x \times \frac{\sin x}{\cos x}$ $= \sin^2 x$ $\frac{dy}{dx} = 2 \sin x \cos x$	$y = x \ln x$ $\frac{dy}{dx} = \ln x + 1$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$ $\frac{dy}{dx} = \left(\frac{x}{3}\right)^3 \times \frac{1}{3\sqrt[3]{x}} + \sqrt[3]{x} \times \frac{x^2}{3}$ $= \frac{x^3}{9\sqrt[3]{x}} + \frac{x^2 \sqrt[3]{x}}{3}$ $= \frac{x^2}{3} \left(\frac{\sqrt[3]{x^2}}{3} + \sqrt[3]{x}\right)$	$y = \sin(x^2) \cos(x^2)$ $\frac{dy}{dx} = 2x[\cos^2(x^2) + \sin^2(x^2)]$	$y = e^{x^2} \ln(x^2)$ $\frac{dy}{dx} = \frac{2e^{x^2}}{x} + 2xe^{x^2} \ln(x^2)$

Quotient Rule

$y = \frac{x+1}{2x+1}$ $\frac{dy}{dx} = \frac{2x+1-2(x+1)}{(2x+1)^2}$ $= \frac{-1}{(2x+1)^2}$	$y = \frac{3x^2}{\sin x}$ $\frac{dy}{dx} = \frac{6x\sin x - 3x^2\cos x}{\sin^2 x}$ $= \frac{3x}{\sin x} \left(2 - \frac{x}{\tan x} \right)$	$y = \frac{2x}{e^x}$ $\frac{dy}{dx} = \frac{2e^x - 2xe^x}{e^{2x}}$ $= \frac{2-2x}{e^x}$
$y = \frac{x^2+1}{x^2-1}$ $\frac{dy}{dx} = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}$ $= \frac{-4x}{(x^2-1)^2}$	$y = \tan x = \frac{\sin x}{\cos x}$ $\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ $= 1 + \frac{\sin^2 x}{\cos^2 x}$ $= \sec^2 x$	$y = \frac{e^x}{e^{-x}} = e^x \times e^x = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$
$y = \frac{x^2}{\sqrt{x}}$ $\frac{dy}{dx} = \frac{2x\sqrt{x} - \frac{x^2}{2\sqrt{x}}}{x}$ $= 2\sqrt{x} - \frac{\sqrt{x}}{2}$	$y = \frac{x^2}{\tan x}$ $\frac{dy}{dx} = \frac{2x\tan x - x^2\sec x}{\tan^2 x}$ $= \frac{2x}{\tan x} - \frac{\tan^2 x \cos x}{x}$ $= \frac{\tan x}{x} \left(2 - \frac{\tan x \cos x}{\tan x} \right)$	$y = \frac{e^{-4x}}{4e^{4x}} = \frac{1}{4}(e^{-4x} \times e^{-4x})$ $= \frac{e^{-8x}}{4}$ $\frac{dy}{dx} = \frac{-2}{e^{8x}}$
$y = \sqrt[3]{\frac{x+1}{x-1}} = \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}}$ $\frac{dy}{dx} = \frac{\frac{1}{3}(x-1)^{\frac{1}{3}}(x+1)^{-\frac{2}{3}} - \frac{1}{3}(x-1)^{-\frac{2}{3}}(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{2}{3}}}$ $= \frac{1}{3} \left[\frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{2}{3}} - \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{4}{3}}} \right]$ $= \frac{1}{3} \left[\frac{(x-1) - (x+1)}{(x+1)^{\frac{2}{3}}(x-1)^{\frac{4}{3}}} \right]$ $= \frac{-2}{3} \left[\frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}} \right]$	$y = \frac{\cot x}{2\sec x} = \frac{\cos x}{\cos x} \div \frac{2}{\cos x}$ $= \frac{\sin x}{\cos x} \times \frac{2}{2}$ $= \frac{\cos^2 x}{2\sin x}$ $\frac{dy}{dx} = \frac{-4\sin^2 x \cos x - 2\cos^3 x}{4\sin^2 x \cos x}$ $= -x\cos x - \frac{2\cos^3 x}{2\sin^2 x}$	$y = \frac{\ln(x^2)}{e^{x^2}}$ $\frac{dy}{dx} = \frac{\frac{2e^{x^2}}{x} - 2xe^{x^2}\ln x^2}{e^{2x^2}}$ $= \frac{\frac{2}{x} - 2x\ln x^2}{e^{x^2}}$