Chain, Product, Quotient Rule Recap

Differentiate these.

Chain Rule

$y = (3x+4)^3$	$y = sin(2x)$	$y = e^{2x}$
$y = (3x^3+1)^3$	$y = sin(x^2)$	$y = e^{(x^3)}$
$y = \frac{4}{4-x^4}$	$y = \frac{1}{\left(\frac{1}{cosecx}\right)}$	$y = ln\left(\frac{1}{e^x}\right)$
$y = \sqrt{\sqrt{x}+1}$	$y = sin\big(cos(tanx)\big)$	$y = e^{(e^x)}$

Product Rule

$y = (3x+4)(2x-3)$	$y = x^2 sinx$	$y = xe^x$
$y = (3x^2+4)(2x^4+3)$	$y = sinxcosx$	$y = x^3e^x$
$y = \sqrt{3x}\sqrt{2x}$	$y = sinxcosxtanx$	$y = xlnx$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$	$y = sin(x^2)cos(x^2)$	$y = e^{x^2}ln(x^2)$

Quotient Rule

$y = \frac{x+1}{2x+1}$	$y = \frac{3x^2}{sinx}$	$y = \frac{2x}{e^x}$
$y = \frac{x^2+1}{x^2-1}$	$y = tanx$	$y = \frac{e^x}{e^{-x}}$
$y = \frac{x^2}{\sqrt{x}}$	$y = \frac{x^2}{tanx}$	$y = \frac{e^{-4x}}{4e^{4x}}$
$y = \sqrt[3]{\frac{x+1}{x-1}}$	$y = \frac{cotx}{2secx}$	$y = \frac{ln(x^2)}{e^{x^2}}$

Chain, Product, Quotient Rule Recap - Answers

Differentiate these.

Chain Rule

$y=(3x+4)^3$ $\frac{dy}{dx}=9(3x+4)^2$	$y=sin(2x)$ $\frac{dy}{dx}=2cos(2x)$	$y=e^{2x}$ $\frac{dy}{dx}=2e^{2x}$
$y=(3x^3+1)^3$ $\frac{dy}{dx}=3(3x^3+1)^2(9x^2)$ $=27x^2(3x^3+1)^2$	$y=sin(x^2)$ $\frac{dy}{dx}=2xcos(x^2)$	$y=e^{(x^3)}$ $\frac{dy}{dx}=3x^2e^{(x^3)}$
$y=\frac{4}{4-x^4}=4(4-x^4)^{-1}$ $\frac{dy}{dx}=-4(4-x^4)^{-2}\times -4x^3$ $=\frac{16x^3}{(4-x^4)^2}$	$y=\frac{1}{\left(\frac{1}{cosecx}\right)}=\frac{1}{sinx}$ $=(sinx)^{-1}$ $\frac{dy}{dx}=-(sinx)^{-2}\times cosx$ $=\frac{-cosx}{(sinx)^2}=cotxcosecx$	$y=ln\left(\frac{1}{e^x}\right)$ $\frac{dy}{dx}=\frac{1}{\frac{1}{e^x}}\times -e^{-x}$ $=-e^x\times e^{-x}$ $=-e^0$ $=1$ (convince yourself that this is the case)
$y=\sqrt{\sqrt{x}+1}=\left(x^{\frac{1}{2}}+1\right)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}\left(x^{\frac{1}{2}}+1\right)^{-\frac{1}{2}}\times\frac{1}{2}x^{-\frac{1}{2}}$ $=\frac{1}{4}\times\frac{1}{\sqrt{\sqrt{x}+1}}\times\frac{1}{\sqrt{x}}$ $=\frac{1}{4\sqrt{x}\sqrt{\sqrt{x}+1}}$	$y=sin(cos(tanx))$ $\frac{dy}{dx}=-cos(cos(tanx))sin(tanx)sec^2x$	$y=e^{e^x}$ $\frac{dy}{dx}=e^{e^x}e^x=e^{e^x+x}$

Product Rule

$y = (3x+4)(2x-3)$ $\frac{dy}{dx} = 2(3x+4) + 3(2x-3)$ $= 12x - 1$	$y = x^2 sinx$ $\frac{dy}{dx} = x^2 cosx + 2xsinx$	$y = xe^x$ $\frac{dy}{dx} = xe^x + e^x$ $= (x+1)e^x$
$y = (3x^2+4)(2x^4+3)$ $\frac{dy}{dx} = 8x^3(3x^2+4) + 6x(2x^4+3)$ $= 2x[4x^2(3x^2+4) + 3(2x^4+3)]$ $= 2x[12x^4 + 16x^2 + 6x^4 + 9]$ $= 2x[18x^4 + 16x^2 + 9]$	$y = sinxcosx$ $\frac{dy}{dx} = cos^2x - sin^2x$	$y = x^3e^x$ $\frac{dy}{dx} = 3x^2e^x + x^3e^x$ $= x^2e^x(x+3)$
$y = \sqrt{3x}\sqrt{2x}$ $= (6x^2)^{\frac{1}{2}} = \sqrt{6}x$ $\frac{dy}{dx} = \sqrt{6}$ (easy method?)	$y = sinxcosxtanx$ $= sinxcosx \times \frac{sinx}{cosx}$ $= sin^2x$ $\frac{dy}{dx} = 2sinxcosx$	$y = xlnx$ $\frac{dy}{dx} = lnx + 1$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$ $\frac{dy}{dx} = \left(\frac{x}{3}\right)^3 \times \frac{1}{3\sqrt[3]{x}} + \sqrt[3]{x} \times \frac{x^2}{3}$ $= \frac{x^3}{9\sqrt[3]{x}} + \frac{x^2\sqrt[3]{x}}{3}$ $= \frac{x^2}{3}\left(\frac{\sqrt[3]{x^2}}{3} + \sqrt[3]{x}\right)$	$y = sin(x^2)cos(x^2)$ $\frac{dy}{dx} = 2x[cos^2(x^2) + sin^2(x^2)]$	$y = e^{x^2} ln(x^2)$ $\frac{dy}{dx} = \frac{2e^{x^2}}{x} + 2xe^{x^2} ln(x^2)$

Quotient Rule

$y=\frac{x+1}{2x+1}$ $\frac{dy}{dx}=\frac{2x+1-2(x+1)}{(2x+1)^2}$ $=\frac{-1}{(2x+1)^2}$	$y=\frac{3x^2}{sinx}$ $\frac{dy}{dx}=\frac{6xsinx-3x^2cosx}{sin^2x}$ $=\frac{3x}{sinx}\left(2-\frac{x}{tanx}\right)$	$y=\frac{2x}{e^x}$ $\frac{dy}{dx}=\frac{2e^x-2xe^x}{e^{2x}}$ $=\frac{2-2x}{e^x}$
$y=\frac{x^2+1}{x^2-1}$ $\frac{dy}{dx}=\frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2}$ $=\frac{-4x}{(x^2-1)^2}$	$y=tanx=\frac{sinx}{cosx}$ $\frac{dy}{dx}=\frac{cos^2x+sin^2x}{cos^2x}$ $=1+sin^2x$ $=sec^2x$	$y=\frac{e^x}{e^{-x}}=e^x\times e^x=e^{2x}$ $\frac{dy}{dx}=2e^{2x}$
$y=\frac{x^2}{\sqrt{x}}$ $\frac{dy}{dx}=\frac{2x\sqrt{x}-\frac{x^2}{2\sqrt{x}}}{x}$ $=2\sqrt{x}-\frac{\sqrt{x}}{2}$	$y=\frac{x^2}{tanx}$ $\frac{dy}{dx}=\frac{2xtanx-x^2secx}{tan^2x}$ $=\frac{2x}{tanx}-\frac{x^2}{tan^2xcosx}$ $=\frac{x}{tanx}\left(2-\frac{x}{tanxcosx}\right)$	$y=\frac{e^{-4x}}{4e^{4x}}=\frac{1}{4}(e^{-4x}\times e^{-4x})$ $=\frac{e^{-8x}}{4}$ $\frac{dy}{dx}=\frac{-2}{e^{8x}}$
$y=\sqrt[3]{\frac{x+1}{x-1}}=\frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}}$ $\frac{dy}{dx}=\frac{\frac{1}{3}(x-1)^{\frac{1}{3}}(x+1)^{\frac{-2}{3}}-\frac{1}{3}(x-1)^{-\frac{2}{3}}(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{2}{3}}}$ $=\frac{1}{3}\left[\frac{(x-1)^{-\frac{1}{3}}}{(x+1)^{\frac{2}{3}}}-\frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{4}{3}}}\right]$ $=\frac{1}{3}\left[\frac{(x-1)-(x+1)}{(x+1)^{\frac{2}{3}}(x-1)^{\frac{4}{3}}}\right]$ $=\frac{-2}{3}\left[\frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}}\right]$	$y=\frac{cotx}{2secx}=\frac{cosx}{sinx}\div\frac{2}{cosx}$ $=\frac{cosx}{sinx}\times\frac{cosx}{2}$ $=\frac{cos^2x}{2sinx}$ $\frac{dy}{dx}=\frac{-4sin^2xcosx-2cos^3x}{4sin^2x}$ $=-xcosx-\frac{cosx}{2tan^2x}$	$y=\frac{ln(x^2)}{e^{x^2}}$ $\frac{dy}{dx}=\frac{\frac{2e^{x^2}}{x}-2xe^{x^2}lnx^2}{e^{2x^2}}$ $=\frac{\frac{2}{x}-2xlnx^2}{e^{x^2}}$

Chain, Product, Quotient Rule Recap

Differentiate these.

Polynomials

$y = (3x+4)^3$	$y = (3x+4)(2x-3)$	$y = \frac{x+1}{2x+1}$
$y = (3x^3+1)^3$	$y = (3x^2+4)(2x^4+3)$	$y = \frac{x^2+1}{x^2-1}$
$y = \frac{4}{4-x^4}$	$y = \sqrt{3x}\sqrt{2x}$	$y = \frac{x^2}{\sqrt{x}}$
$y = \sqrt{\sqrt{x}+1}$	$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$	$y = \sqrt[3]{\frac{x+1}{x-1}}$

Trigonometry

$y = sin(2x)$	$y = x^2 sinx$	$y = \frac{3x^2}{sinx}$
$y = sin(x^2)$	$y = sinxcosx$	$y = tanx$
$y = \frac{1}{\left(\frac{1}{cosecx}\right)}$	$y = sinxcosxtanx$	$y = \frac{x^2}{tanx}$
$y = sin\left(cos(tanx)\right)$	$y = sin(x^2)cos(x^2)$	$y = \frac{cotx}{2secx}$

Exponentials

$y = e^{2x}$	$y = xe^x$	$y = \frac{2x}{e^x}$
$y = e^{(x^3)}$	$y = x^3e^x$	$y = \frac{e^x}{e^{-x}}$
$y = ln\left(\frac{1}{e^x}\right)$	$y = xlnx$	$y = \frac{e^{-4x}}{4e^{4x}}$
$y = e^{(e^x)}$	$y = e^{x^2}ln(x^2)$	$y = \frac{ln(x^2)}{e^{x^2}}$

Build It Up Practice

$y = (2x+3)^7$	$y = 2x(4x+5)$	$y = \frac{2x}{4x+5}$
$y = (2x^2+3)^7$	$y = 2x^2(4x+5)$	$y = \frac{2x^2}{4x+5}$
$y = (2x^2+3x)^7$	$y = (2x+3)(4x+5)$	$y = \frac{2x+3}{4x+5}$
$y = (2x^3+3x^2)^7$	$y = (2x^2+3x)(4x+5)$	$y = \frac{2x^2+3x}{4x+5}$
$y = \frac{1}{2x+3}$	$y = \frac{1}{x}(4x+5)$	$y = \frac{1}{4x+5}$
$y = \frac{1}{2x^2+3x}$	$y = \sqrt{x}(4x+5)$	$y = \frac{\sqrt{x}}{\sqrt{x+1}}$
$y = sin(2x)$	$y = \sin x \cos x$	$y = \frac{1}{\sqrt{x}\sqrt{x+1}}$
$y = sin(2x^2+3)$	$y = \sin 2x \cos 2x$	$y = \frac{\sin x}{\cos x}$
$y = \cos(2x^2+3)$	$y = \sin(2x+3)\cos(4x+5)$	$y = \frac{\sin 2x}{\cos 2x}$
$y = sin\left(\frac{1}{2x+3}\right)$	$y = (2x+3)^7(4x+5)$	$y = \left(\frac{2x+3}{4x+5}\right)^7$

Build It Up Practice - Answers

$y=(2x+3)^7,\ \frac{dy}{dx}=$	$y=2x(4x+5),\ \frac{dy}{dx}=$	$y=\frac{2x}{4x+5},\ \frac{dy}{dx}=$
$y=(2x^2+3)^7,\ \frac{dy}{dx}=$	$y=2x^2(4x+5),\ \frac{dy}{dx}=$	$y=\frac{2x^2}{4x+5},\ \frac{dy}{dx}=$
$y=(2x^2+3x)^7,\ \frac{dy}{dx}=$	$y=(2x+3)(4x+5),\ \frac{dy}{dx}=$	$y=\frac{2x+3}{4x+5},\ \frac{dy}{dx}=$
$y=(2x^3+3x^2)^7,\ \frac{dy}{dx}=$	$y=(2x^2+3x)(4x+5),\ \frac{dy}{dx}=$	$y=\frac{2x^2+3x}{4x+5},\ \frac{dy}{dx}=$
$y=\frac{1}{2x+3},\ \frac{dy}{dx}=$	$y=\frac{1}{x}(4x+5),\ \frac{dy}{dx}=$	$y=\frac{1}{4x+5},\ \frac{dy}{dx}=$
$y=\frac{1}{2x^2+3x},\ \frac{dy}{dx}=$	$y=\sqrt{x}(4x+5),\ \frac{dy}{dx}=$	$y=\frac{\sqrt{x}}{\sqrt{x+1}},\ \frac{dy}{dx}=$
$y=sin(2x),\ \frac{dy}{dx}=$	$y=\sin x\cos x,\ \frac{dy}{dx}=$	$y=\frac{1}{\sqrt{x}\sqrt{x+1}},\ \frac{dy}{dx}=$
$y=sin(2x^2+3),\ \frac{dy}{dx}=$	$y=\sin 2x\cos 2x,\ \frac{dy}{dx}=$	$y=\frac{\sin x}{\cos x},\ \frac{dy}{dx}=$
$y=\cos(2x^2+3),\ \frac{dy}{dx}=$	$y=\sin(2x+3)\cos(4x+5),\ \frac{dy}{dx}=$	$y=\frac{\sin 2x}{\cos 2x},\ \frac{dy}{dx}=$
$y=sin\left(\frac{1}{2x+3}\right),\ \frac{dy}{dx}=$	$y=(2x+3)^7(4x+5),\ \frac{dy}{dx}=$	$y=\left(\frac{2x+3}{4x+5}\right)^7,\ \frac{dy}{dx}=$

Three of Each

$y = (2x^3 - 7)^4$	$y = (3x+1)\left(\sqrt{3x}+1\right)$	$f(x) = \dfrac{x}{5x-6}$
$f(x) = \dfrac{1}{2}e^{2x^3-7}$	$y = \mathrm{e}^{3\mathrm{x}} ln(3\mathrm{x})$	$y = \dfrac{ln(5x)}{e^{5x}}$
$y = ln[sin(2x^3 - 7)]$	$f(x) = sin(3x)cos(3x)$	$y = \dfrac{sin5x}{tan5x}$

Three of Each - Answers

$y = (2x^3 - 7)^4$	$y = (3x + 1)\left(\sqrt{3x} + 1\right)$	$f(x) = \dfrac{x}{5x - 6}$
$f(x) = \dfrac{1}{2}e^{2x^3 - 7}$	$y = \mathrm{e}^{3\mathrm{x}} ln(3\mathrm{x})$	$y = \dfrac{ln(5x)}{e^{5x}}$
$y = ln[sin(2x^3 - 7)]$	$f(x) = sin(3x)cos(3x)$	$y = \dfrac{sin5x}{tan5x}$