Binomial Theorem

$$(1+x)^n = 1+\frac{n}{1}x+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3\ldots$$

n must be rational and not positive integer

$|x|<1$

Questions

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5}$
2) $(1+2x)^{-5}$
3) $(4+x)^{-5}$
4) $(4+12x)^{-5}$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{1/2}$
6) $(1+2x)^{1/2}$
7) $(4+x)^{1/2}$
8) $(4+12x)^{1/2}$

9) $(1+x)^{-3}+(2+3x)^{-3}$

Extra Practice

As far as the x^4 term

10) $(1+x)^{-6}$
11) $(1+2x)^{-6}$
12) $(4+3x)^{-5}$

As far as the x^3 term

13) $(1+x)^{1/4}$
14) $(1+2x)^{1/4}$
15) $(4+12x)^{1/4}$
16) $(4+x)^{1/4}$

Application

Show that $\left(1+\frac{x}{25}\right)^{\frac{1}{2}} = 1+\frac{x}{50}-\frac{x^2}{5000}+\frac{x^3}{250000}-\ldots$

By substituting $x=1$ into the expression above, deduce that $\sqrt{26}\approx 5.09902$

Find an approximation for $\sqrt{28}$.

Why can't this expansion be used to find an approximation for $\sqrt{35}$?

Answers

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5} = 1-5x+15x^2-35x^3+70x^4$

2) $(1+2x)^{-5} = 1-10x+60x^2-280x^3+1120x^4$

3) $(4+x)^{-5} = 4^{-5}(1+\frac{x}{4})^{-5} = \frac{1}{1024}-\frac{5}{4096}x+\frac{15}{16384}x^2-\frac{35}{65536}x^3+\frac{35}{131072}x^4$

4) $(4+12x)^{-5} = 4^{-5}(1+3x)^{-5} = \frac{1}{1024}-\frac{15}{1024}x+\frac{135}{1024}x^2-\frac{945}{1024}x^3+\frac{2835}{512}x^4$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{1/2} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$

6) $(1+2x)^{1/2} = 1+x-\frac{1}{2}x^2+\frac{1}{2}x^3$

7) $(4+x)^{1/2} = 4^{1/2}\left[(1+\frac{x}{4})^{1/2}\right] = 2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3$

8) $(4+12x)^{1/2} = 4^{1/2}\left[(1+3x)^{1/2}\right] = 2+3x-\frac{9}{4}x^2+\frac{27}{8}x^3$

9) $(1+x)^{-3}+(2+3x)^{-3} = \frac{9}{8}-\frac{57}{16}x+\frac{123}{16}x^2-\frac{455}{32}x^3$

Extra practice:

10) $(1+x)^{-6} = 1-6x+21x^2-56x^3+126x^4$

11) $(1+2x)^{-6} = 1-12x+84x^2-448x^3+2016x^4$

12) $(4+3x)^{-5} = 4^{-5}\left[(1+\frac{3}{4}x)^{-5}\right] = \frac{1}{1024}-\frac{15}{4096}x+\frac{135}{16384}x^2-\frac{945}{65536}x^3+\frac{2835}{262144}x^4$

13) $(1+x)^{1/4} = 1+\frac{1}{4}x-\frac{3}{32}x^2+\frac{7}{128}x^3$

14) $(1+2x)^{1/4} = 1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{7}{16}x^3$

15) $(4+12x)^{1/4} = \sqrt{2}+\frac{3\sqrt{2}}{4}x-\frac{27\sqrt{2}}{32}x^2+\frac{189\sqrt{2}}{128}x^3$

16) $(4+x)^{1/4} = \sqrt{2}+\frac{\sqrt{2}}{16}x-\frac{3\sqrt{2}}{512}x^2+\frac{7\sqrt{2}}{8192}x^3$

$$
\begin{array}{r}
1.\ 2\ 3\ 4\ 5\ 6\ 7\ 9\ 0\ 1\ 2\ldots \\
81\overline{\big)\,100.{}^{19}0\,{}^{28}0\,{}^{37}0\,{}^{46}0\,{}^{55}0\,{}^{64}0\,{}^{73}0\,{}^{1}0\,{}^{10}0\,{}^{19}0\ldots.}
\end{array}
$$

Activity…

$$(1+x)^n = 1+\frac{n}{1}x+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3\ldots$$

Check for…

$n=-1\,, x=-x$ Compare LHS vs RHS Compare Graphs (between limits) Compare the geometric series Substitute values of x to check ($x=1, x={}^{1}/_{2}$)
$n=-2, \quad x=-x$ Compare LHS vs RHS Compare Graphs (between limits) Compare the geometric series Substitute values of x to check ($x=1, x={}^{1}/_{2}, x=0.1$) Relate to 100/81
$n={}^{1}/_{2}, \quad x=x$ Compare LHS vs RHS Compare Graphs (between limits) Compare the geometric series Substitute values of x to check Square both sides