

Binomial Theorem

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$$

n must be rational and not positive integer

$$|x| < 1$$

Questions

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5}$

3) $(4+x)^{-5}$

2) $(1+2x)^{-5}$

4) $(4+12x)^{-5}$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{\frac{1}{2}}$

7) $(4+x)^{\frac{1}{2}}$

6) $(1+2x)^{\frac{1}{2}}$

8) $(4+12x)^{\frac{1}{2}}$

9) $(1+x)^{-3} + (2+3x)^{-3}$

Extra Practice

As far as the x^4 term

10) $(1+x)^{-6}$

11) $(1+2x)^{-6}$

12) $(4+3x)^{-5}$

As far as the x^3 term

13) $(1+x)^{\frac{1}{4}}$

14) $(1+2x)^{\frac{1}{4}}$

15) $(4+12x)^{\frac{1}{4}}$

16) $(4+x)^{\frac{1}{4}}$

Application

Show that $\left(1 + \frac{x}{25}\right)^{\frac{1}{2}} = 1 + \frac{x}{50} - \frac{x^2}{5000} + \frac{x^3}{250000} - \dots$

By substituting $x=1$ into the expression above, deduce that $\sqrt{26} \approx 5.09902$

Find an approximation for $\sqrt{28}$.

Why can't this expansion be used to find an approximation for $\sqrt{35}$?

Answers

Rewrite as approximations using the binomial expansion as far as the x^4 term:

$$1) (1+x)^{-5} = 1 - 5x + 15x^2 - 35x^3 + 70x^4$$

$$2) (1+2x)^{-5} = 1 - 10x + 60x^2 - 280x^3 + 1120x^4$$

$$3) (4+x)^{-5} = 4^{-5} \left(1 + \frac{x}{4}\right)^{-5} = \frac{1}{1024} - \frac{5}{4096}x + \frac{15}{16384}x^2 - \frac{35}{65536}x^3 + \frac{35}{131072}x^4$$

$$4) (4+12x)^{-5} = 4^{-5} (1+3x)^{-5} = \frac{1}{1024} - \frac{15}{1024}x + \frac{135}{1024}x^2 - \frac{945}{1024}x^3 + \frac{2835}{512}x^4$$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

$$5) (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$$

$$6) (1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$$

$$7) (4+x)^{\frac{1}{2}} = 4^{\frac{1}{2}} \left[\left(1 + \frac{x}{4}\right)^{\frac{1}{2}} \right] = 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3$$

$$8) (4+12x)^{\frac{1}{2}} = 4^{\frac{1}{2}} \left[(1+3x)^{\frac{1}{2}} \right] = 2 + 3x - \frac{9}{4}x^2 + \frac{27}{8}x^3$$

$$9) (1+x)^{-3} + (2+3x)^{-3} = \frac{9}{8} - \frac{57}{16}x + \frac{123}{16}x^2 - \frac{455}{32}x^3$$

Extra practice:

$$10) (1+x)^{-6} = 1 - 6x + 21x^2 - 56x^3 + 126x^4$$

$$11) (1+2x)^{-6} = 1 - 12x + 84x^2 - 448x^3 + 2016x^4$$

$$12) (4+3x)^{-5} = 4^{-5} \left[\left(1 + \frac{3}{4}x\right)^{-5} \right] = \frac{1}{1024} - \frac{15}{4096}x + \frac{135}{16384}x^2 - \frac{945}{65536}x^3 + \frac{2835}{262144}x^4$$

$$13) (1+x)^{\frac{1}{4}} = 1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$$

$$14) (1+2x)^{\frac{1}{4}} = 1 + \frac{1}{2}x - \frac{3}{8}x^2 + \frac{7}{16}x^3$$

$$15) (4+12x)^{\frac{1}{4}} = \sqrt{2} + \frac{3\sqrt{2}}{4}x - \frac{27\sqrt{2}}{32}x^2 + \frac{189\sqrt{2}}{128}x^3$$

$$16) (4+x)^{\frac{1}{4}} = \sqrt{2} + \frac{\sqrt{2}}{16}x - \frac{3\sqrt{2}}{512}x^2 + \frac{7\sqrt{2}}{8192}x^3$$

$$\begin{array}{r}
 1. \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 9 \ 0 \ 1 \ 2 \ \dots \\
 \hline
 81 \overline{) 100. \overset{19}{0} \overset{28}{0} \overset{37}{0} \overset{46}{0} \overset{55}{0} \overset{64}{0} \overset{73}{0} \overset{10}{0} \overset{19}{0} \overset{0}{0} \dots}
 \end{array}$$

Activity...

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots$$

Check for...

$n = -1, x = -x$
Compare LHS vs RHS $y = \frac{1}{(1-x)}, y = 1 + x + x^2 + x^3 + \dots$
Compare Graphs (between limits)
Compare the geometric series $a = 1, r = x$
Substitute values of x to check ($x = 1, x = 1/2$)
$n = -2, \quad x = -x$
Compare LHS vs RHS $y = \frac{1}{(1-x)^2}, y = 1 + 2x + 3x^2 + 4x^3 + \dots$
Compare Graphs (between limits)
Compare the geometric series $a = 1, r = ?$
Substitute values of x to check ($x = 1, x = 1/2, x = 0.1$)
$x = 0.1 \Rightarrow y = 1 + 0.2 + 0.03 + 0.004 \dots$
Relate to 100/81 (and consider the $10x^9$ term)
$n = 1/2, \quad x = x$
Compare LHS vs RHS $y = \frac{1}{\sqrt{1-x}}, y = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots$
Compare Graphs (between limits)
Compare the geometric series $a = 1, r = \frac{x}{4}$
Substitute values of x to check
Square both sides

Coefficient	x^n	Term	Sum
1	1	1	1
2	0.1	0.2	1.2
3	0.01	0.03	1.23
4	0.001	0.004	1.234
5	0.0001	0.0005	1.2345
6	0.00001	0.00006	1.23456
7	0.000001	0.000007	1.234567
8	0.0000001	0.0000008	1.2345678
9	0.00000001	0.00000009	1.23456789
10	0.000000001	0.0000000 <u>10</u>	1.2345679
		1.234567 910...	