Trigonometrical Integration

1) Using the substitution $u = \cos x$, try (and fail) to integrate $\int \cos^2 x \, dx$.

You cannot complete this because the required derivative is missing from the integrand. Using $u = \cos x$, the derivative is $\frac{du}{dx} = -\sin x$ and there is no $\sin x$ in the integrand to balance or replace dx.

$\cos 2x = 2\cos^2 x - 1$ $\quad$ $\cos 2x = 1 - 2\sin^2 x$ $\quad$ $\sin 3x = 3\sin x - 4\sin^3 x$ $\quad$ $\cos 3x = 4\cos^3 x - 3\cos x$

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2) Use the formulae above to rearrange and then integrate the expressions below.

1) $\int \cos^2 x \, dx$

2) $\int 4\cos^2 x - 3\sin^3 x \, dx$

3) $\int \cos^2 3x \, dx$

4) $\int_0^{\frac{\pi}{4}} 4\cos^2 3x + \sin^3 2x \, dx$

5) $\int \frac{\cos^3 x}{1 + \sin x} dx$

6) $\int x\cos^2 x \, dx$

7) $\int x\sin^2 2x \, dx$

8) $\int \cos^2 x \sin x \, dx$

9) $\int \frac{\sin^3 x}{\cos^2 x} dx$

3) A curve has equation $y = a\cos^2 x + b\sin^2 x$, where a and b are constants. The area under the curve between $x = 0$ and $x = \pi$ is 5π. Given that the curve passes through $(0, 4)$, find the values of a and b.

4) A particle moves in a straight line with velocity $v = 3\sin^2 t + 2\cos^2 t$. Given that its displacement from the origin is zero when $t = 0$, find its displacement at $t = \pi$.

Trigonometrical Integration - answers

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$1 + \cos 2x = 2\cos^2 x$ $\quad$ $1 - \cos 2x = 2\sin^2 x$ $\quad$ $\sin 3x = 3\sin x - 4\sin^3 x$ $\quad$ $\cos 3x = 4\cos^3 x - 3\cos x$

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