UKMT Prime Number Questions

(Answers follow after all the questions)

2005… (none)

2006… (none)

2007…

16. Damien wishes to find out if 457 is a prime number. In order to do this he needs to check whether it is exactly divisible by some prime numbers. What is the smallest number of possible prime number divisors that Damien needs to check before he can be sure that 457 is a prime number?

A 8 B 9 C 10 D 11 E 12

2008…

19. How many prime numbers p are there such that $199p + 1$ is a perfect square?

A 0 B 1 C 2 D 4 E 8

2009… (none)

2010…

15. What is the smallest prime number that is equal to the sum of two prime numbers and is also equal to the sum of three different prime numbers?

A 7 B 11 C 13 D 17 E 19

2011…

12. The *primorial* of a number is the product of all of the prime numbers less than or equal to that number. For example, the primorial of 6 is $2 \times 3 \times 5 = 30$. How many different whole numbers have a primorial of 210?

A 1 B 2 C 3 D 4 E 5

2012…

1. Which of the following cannot be written as the sum of two prime numbers?

   A 5 B 7 C 9 D 10 E 11

2013…

15. For how many positive integers n is $4^n - 1$ a prime number?

    A 0 B 1 C 2 D 3 E infinitely many

2014…

11. A Mersenne prime is a prime of the form $2^p - 1$, where p is also a prime. One of the following is **not** a Mersenne prime. Which one is it?

    A $2^2 - 1$ B $2^3 - 1$ C $2^5 - 1$ D $2^7 - 1$ E $2^{11} - 1$

2015…

23. Given four different non-zero digits, it is possible to form 24 different four-digit numbers containing each of these four digits. What is the largest prime factor of the sum of the 24 numbers?

    A 23 B 93 C 97 D 101 E 113

2016… (none)

UKMT Prime Number Answers

2005… (none)

2006… (none)

2007…

16. A The smallest number of possible prime divisors of 457 that Damien needs to check is the number of prime numbers less than or equal to the square root of 457. Since $21^2 < 457 < 22^2$, he needs to check only primes less than 22. These primes are 2, 3, 5, 7, 11, 13, 17 and 19.

2008…

19. B Let $199p + 1 = X^2$. Then $199p = X^2 - 1 = (X+1)(X-1)$. Note that 197 is prime. If p is also to be prime then **either** $X + 1 = 199$, in which case $X - 1 = 197$, **or** $X - 1 = 199$, in which case $X + 1 = 201$ (and $201 = 3 \times 67$ is not prime). Note that $X - 1 = 1, X + 1 = 199p$ is impossible. Hence $p = 197$ is the only possibility.

2009… (none)

2010…

15. E The first eight prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19.
If the sum of two prime numbers is prime, one of them must be 2.
If the sum of three different prime numbers is prime they must all be odd. The answer is therefore 19 as: $2 + 17 = 19$ and $3 + 5 + 11 = 19$.

2011…

12. D The primorial of 7 is $2 \times 3 \times 5 \times 7 = 210$. As 8, 9 and 10 are not prime numbers, they also have a primorial of 210. The primorial of 11 is $2 \times 3 \times 5 \times 7 \times 11 = 2310$. Hence there are exactly four different whole numbers which have a primorial of 210.

2012…

1. E If an odd number is written as the sum of two prime numbers then one of those primes is 2, since 2 is the only even prime. However, 9 is not prime so 11 cannot be written as the sum of two primes. Note that $5 = 2 + 3$; $7 = 2 + 5$; $9 = 2 + 7$; $10 = 3 + 7$, so 11 is the only alternative which is not the sum of two primes.

2013…

15. B A prime number has exactly two factors, one of which is 1. The expression $4^n - 1$ can be factorised as $4^n - 1 = (2^n + 1)(2^n - 1)$. For $4^n - 1$ to be prime, the smaller of the factors, $2^n - 1$, must equal 1.
If $2^n - 1 = 1$ then $2^n = 2$ giving $n = 1$. So there is exactly one value of n for which $4^n - 1$ is prime and this value is 1.

2014…

11. **E** First note that the exponent in each of the five options is prime, so we need to see which of the five numbers is not prime. By direct calculation the numbers are 3, 7, 31, 127 and 2047. Only the last number is not prime, as $2047 = 23 \times 89$.

2015…

23. **D** Let a four-digit positive integer be expressed as $1000a + 100b + 10c + d$ where a, b, c and d are all different. In the 24 possible permutations of a, b, c and d, each of the four letters appears in each position six times. Adding all 24 numbers together gives $1000(6a+6b+6c+6d)+100(6a+6b+6c+6d)+10(6a+6b+6c+6d)+6a+6b+6c+6d$. The total is therefore $1111 \times 6(a+b+c+d)$ which factorises to $2 \times 3 \times 11 \times 101(a+b+c+d)$. As $a+b+c+d < 101$, the largest prime factor of the sum is 101.

2016… (none)