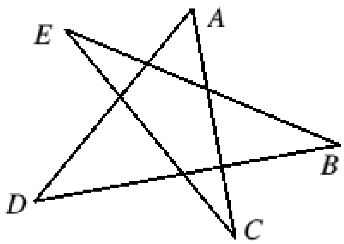
UKMT Angles Questions
(Answers follow after all the questions)

2005…

13. In the figure shown, what is the sum of the interior angles at A, B, C, D, E?

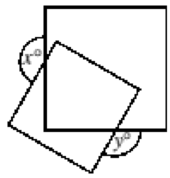
A $90°$ B $135°$ C $150°$ D $180°$
E more information required.



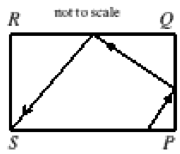
2006…

3. The diagram shows overlapping squares.
What is the value of $x + y$?

A 270 B 300 C 330
D 360 E more information needed



21. A toy pool table is 6 feet long and 3 feet wide. It has pockets at each of the four corners P, Q, R and S. When a ball hits a side of the table, it bounces off the side at the same angle as it hit that side. A ball, initially 1 foot to the left of pocket P, is hit from the side SP towards the side PQ as shown.



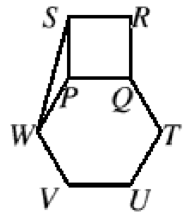
How many feet from P does the ball hit side PQ if it lands in pocket S after two bounces?

A 1 B $\frac{6}{7}$ C $\frac{3}{4}$ D $\frac{2}{3}$ E $\frac{3}{5}$

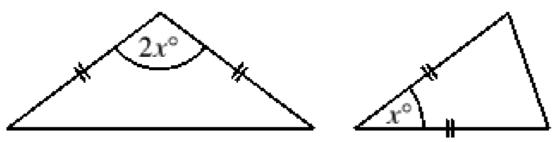
2007…

4. The diagram shows square $PQRS$ and regular hexagon $PQTUVW$.
What is the size of $\angle PSW$?

A $10°$ B $12°$ C $15°$ D $24°$ E $30°$



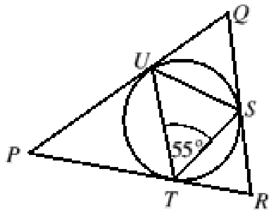
17. The two triangles have equal areas and the four marked lengths are equal.
What is the value of x?



A 30 B 45 C 60 D 75 E more information needed

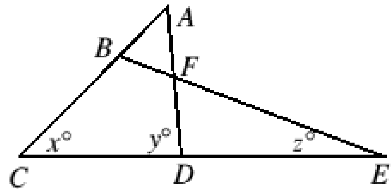
19. The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U, as shown in the diagram. The size of $\angle STU = 55°$. What is the size of $\angle PQR$?

A 55° B 60° C 65° D 70° E 75°



2008…

8. In the figure shown, $AB = AF$ and ABC, AFD, BFE and CDE are all straight lines. Which of the following expressions gives z in terms of x and y?

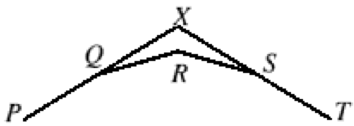


A $\frac{y-x}{2}$ B $y-\frac{x}{2}$ C $\frac{y-x}{3}$ D $y-\frac{x}{3}$ E $y-x$

2009…

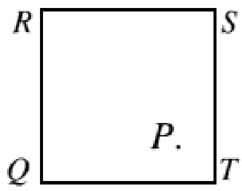
14. P, Q, R, S, T are vertices of a regular polygon. The sides PQ and TS are produced to meet at X, as shown in the diagram, and $\angle QXS = 140°$. How many sides does the polygon have?

A 9 B 18 C 24 D 27 E 40



20. A point P is chosen at random inside a square $QRST$. What is the probability that $\angle RPQ$ is acute?

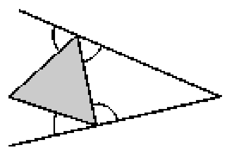
A $\frac{3}{4}$ B $\sqrt{2}-1$ C $\frac{1}{2}$ D $\frac{\pi}{4}$ E $1-\frac{\pi}{8}$



2010…

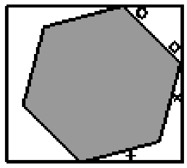
3. The diagram shows an equilateral triangle touching two straight lines. What is the sum of the four marked angles?

A 120° B 180° C 240° D 300° E 360°

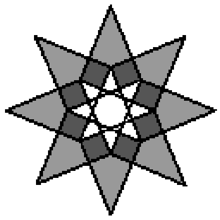


2011…

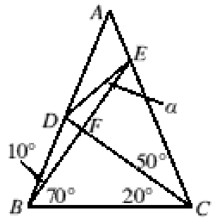
5. The diagram shows a regular hexagon inside a rectangle.
What is the sum of the four marked angles?
A 90° B 120° C 150° D 180° E 210°



19. The diagram shows a small regular octagram (an eight-sided star) surrounded by eight squares (dark grey) and eight kites (light grey) to make a large regular octagram. Each square has area 1.
What is the area of one of the light grey kites?
A 2 B $\sqrt{2}+1$ C $\frac{21}{8}$ D $4\sqrt{2}-3$ E $\frac{11}{4}$

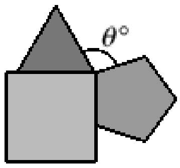


22. In the diagram, $\angle ABE = 10°$; $\angle EBC = 70°$; $\angle ACD = 50°$; $\angle DCB = 20°$; $\angle DEF = \alpha$.
Which of the following is equal to $\tan\alpha$?
A $\dfrac{\tan 10° \tan 20°}{\tan 50°}$ B $\dfrac{\tan 10° \tan 20°}{\tan 70°}$ C $\dfrac{\tan 10° \tan 50°}{\tan 70°}$
D $\dfrac{\tan 20° \tan 50°}{\tan 70°}$ E $\dfrac{\tan 10° \tan 70°}{\tan 50°}$



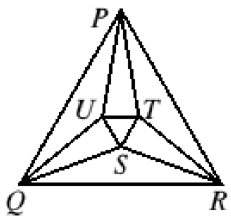
2012…

2. The diagram shows an equilateral triangle, a square and a regular pentagon which all share a common vertex. What is the value of θ?
A 98 B 102 C 106 D 110 E 112

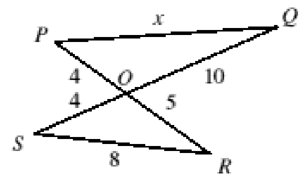


2013…

17. The equilateral triangle PQR has side-length 1. The lines PT and PU trisect the angle RPQ, the lines RS and RT trisect the angle QRP and the lines QS and QU trisect the angle PQR.
What is the side-length of the equilateral triangle STU?
A $\dfrac{\cos 80°}{\cos 20°}$ B $\frac{1}{3}\cos 20°$ C $\cos^2 20°$
D $\frac{1}{6}$ E $\cos 20° \cos 80°$



24. The diagram shows two straight lines PR and QS crossing at O.
What is the value of x?



A $7\sqrt{2}$ B $2\sqrt{29}$ C $14\sqrt{2}$ D $7(1+\sqrt{13})$ E $9\sqrt{2}$

2014…

25. Figure 1 shows a tile in the form of a trapezium, where $\alpha = 83\frac{1}{3}°$. Several copies of the tile are placed together to form a symmetrical pattern, part of which is shown in Figure 2. The outer border of the complete pattern is a regular 'star polygon'. Figure 3 shows an example of a regular 'star polygon'.

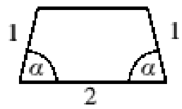
Figure 1


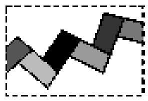
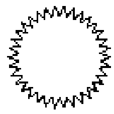
Figure 2


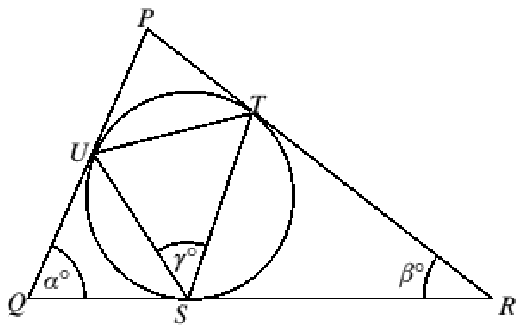
Figure 3


How many tiles are there in the complete pattern?

A 48 B 54 C 60 D 66 E 72

2015…

12. A circle touches the sides of triangle PQR at the points S, T and U as shown. Also $\angle PQR = \alpha°$, $\angle PRQ = \beta°$ and $\angle TSU = \gamma°$. Which of the following gives γ in terms of α and β?



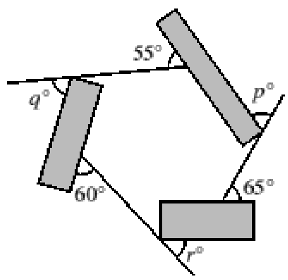
A $\frac{1}{2}(\alpha + \beta)$ B $180 - \frac{1}{2}(\alpha + \beta)$
C $180 - (\alpha + \beta)$ D $\alpha + \beta$
E $\frac{1}{3}(\alpha + \beta)$

2016…

4. Alex draws a scalene triangle. One of the angles is 80°.
Which of the following could be the difference between the other two angles in Alex's triangle?

A 0° B 60° C 80° D 100° E 120°

15. The diagram shows three rectangles and three straight lines.
What is the value of $p + q + r$?

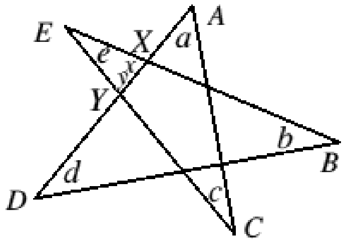


A 135 B 180 C 210
D 225 E 270

UKMT Angles Answers

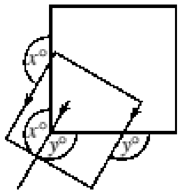
2005…

13. **D** Let a, b, c, d, e, x and y represent the sizes in degrees of certain angles in the figure, as shown and let the points of intersection of AD with EB and EC be X and Y respectively. Angle EXY is an exterior angle of triangle XBD so $x = b + d$. Similarly, angle EYX is an exterior angle of triangle YAC so $y = a + c$. In triangle EXY, $e + x + y = 180$, so $a + b + c + d + e = 180$.

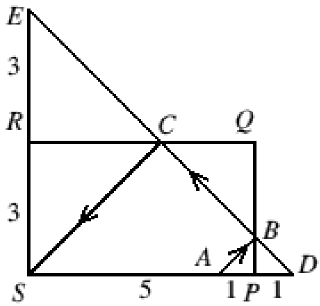


2006…

3. **A** The overlapping region of the two squares is bounded by a pentagon. Two of the interior angles of this pentagon are vertically opposite the given angles of size $x°$ and $y°$, whilst the other three interior angles are all right angles. So $x + y + 3 \times 90 = 540$, that is $x + y = 270$.
(*The diagram on the right also demonstrates that* $x + y = 270$.)



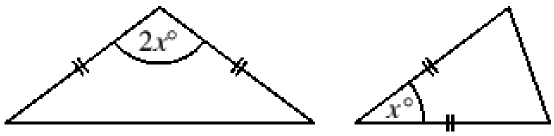
21. **B** The route of the ball is $A \rightarrow B \rightarrow C \rightarrow S$. The diagram also shows point D, the reflection of point A in PQ, and point E, the reflection of point S in QR. As the ball bounces off a side at the same angle at which it hits that side, points D, B, C, E lie in a straight line. Triangles DPB and DSE are similar since both are right-angled and they have a common angle at D. So $\frac{BP}{PD} = \frac{ES}{SD} = \frac{6}{7}$. Hence $BP = \frac{6}{7}$.



2007…

4. **C** $\angle WPQ = 120°$ (interior angle of a regular hexagon), so $\angle WPS = (360 - 120 - 90)° = 150°$. Now $PW = PQ$ (sides of a regular hexagon) and $PS = PQ$ (sides of a square) so $PW = PS$. Therefore triangle PSW is isosceles and $\angle PSW = (180 - 150)° \div 2 = 15°$.

17. **C** Let the equal sides have length k. The height of the triangle on the left is $k \cos x°$ and its base is $2k \sin x°$, so its area is $k^2 \sin x° \cos x°$. The height of the triangle on the right is $k \sin x°$ and its base is k, so its area is $\frac{1}{2}k^2 \sin x°$. Hence $\cos x° = \frac{1}{2}$ and so $x = 60$.

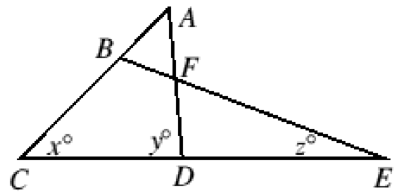


(*Alternatively, the formula* $\Delta = \frac{1}{2}ab \sin C$ *can be used to show that* $\sin x° = \sin 2x°$; *hence* $x + 2x = 180$.)

19. **D** By the Alternate Segment Theorem $\angle QUS = 55°$. Tangents to a circle from an exterior point are equal, so $QU = QS$ and hence $\angle QSU = \angle QUS = 55°$. So $\angle PQR = 180° - 2 \times 55° = 70°$.

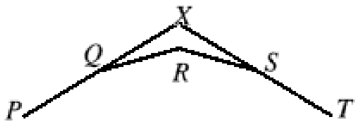
2008…

8. **A** In triangle ACD, $\angle CAD = (180 - x - y)°$.
As $AB = AF$, triangle ABF is isosceles hence $\angle ABF = \angle AFB = \frac{1}{2}(x + y)°$.
Thus $\angle DFE = \angle AFB = \frac{1}{2}(x + y)°$ (vertically opposite angles). Now in triangle DFE, $\angle FDE = (180 - y)°$. Hence $z° = 180° - \angle DFE - \angle FDE = \frac{1}{2}(y - x)°$.

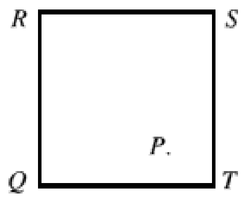


2009…

14. **D** Let the external angle of the regular polygon be $x°$.
Hence $\angle XQR = \angle XSR = x°$ and reflex angle $\angle QRS = (180 + x)°$.
As the sum of the angles in the quadrilateral $QRSX$ is $360°$ then $140 + x + x + 180 + x = 360$.
Hence $3x = 40$ and the polygon has $\frac{360}{40 \div 3} = 27$ sides.



20. **E** If $\angle RPQ = 90°$ then P lies on a semicircle of diameter RQ.
Let x be the side-length of the square $QRST$.
Hence the area of the semicircle $RPQ = \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = \frac{1}{8}\pi x^2$ and the area of square $QRST$ is x^2.
$\angle RPQ$ is acute when P is outside the semicircle RPQ.
Hence the probability that $\angle RPQ$ is acute is $\frac{x^2 - \frac{1}{8}\pi x^2}{x^2} = 1 - \frac{\pi}{8}$.



2010…

3. **C** The three internal angles of an equilateral triangle are all 60°. As the sum of the angles on a straight line is 180° then the sum of the four marked angles is $2 \times (180 - 60)° = 2 \times 120° = 240°$.

2011…

5. **B** As the sum of the angles in a triangle is 180° and all four angles in a rectangle are 90°, the sum of the two marked angles in the triangle is $180° - 90° = 90°$.
Each interior angle of a regular hexagon is 120° and the sum of the angles in a quadrilateral is 360°; hence the sum of the two marked angles in the quadrilateral is $360° - 90° - (360° - 120°) = 30°$.
Hence the sum of the four marked angles is $90° + 30° = 120°$.

19. B As each square has area 1 its side length must be 1.
The external angle of the small regular octagon is $\frac{1}{8} \times 360^\circ = 45^\circ$.
Hence, as the sum of the angles on a straight line is 180° and the sum of the angles in a kite is 360°, the four angles in each of the eight kites (white) are: 90°, 90°, 135° and 45°.
As the light grey kites and the white kites are similar, the interior angles are the same. Two of the sides of the grey kite have length 1. Let the other sides have length a. Using the Cosine Rule twice within a light grey kite, the square of the short diagonal is $1^2 + 1^2 - 2 \times 1 \times 1 \cos 135^\circ = a^2 + a^2 - 2a \times a \cos 45^\circ$. Hence $2 + 2 \times 1/\sqrt{2} = 2a^2 - 2a^2 \times 1/\sqrt{2}$.

Thus $a^2 = \dfrac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \dfrac{\sqrt{2} + 1}{\sqrt{2} - 1}$ and so $a = \sqrt{2} + 1$.

But the area of one of the light grey kites is $2 \times \frac{1}{2}a \times 1 = a$.
Hence the area of one of the light grey kites is $\sqrt{2} + 1$.

22. A As the sum of the angles in a triangle is 180°, in triangle CBF, $\angle BFC = 90^\circ$. As vertically opposite angles are equal $\angle DFE = \angle BFC = 90^\circ$. As the sum of the angles on a straight line is 180°, $\angle DFB = \angle EFC = 90^\circ$. Hence in triangle EFD, $\tan \alpha = \frac{DF}{EF}$: in triangle DFB, $\tan 10^\circ = \frac{DF}{FB}$: in triangle BFC, $\tan 20^\circ = \frac{FB}{FC}$ and in triangle CEF, $\tan 50^\circ = \frac{EF}{FC}$. Thus $\tan \alpha = \frac{DF}{EF} = \frac{\tan 10^\circ FB}{EF} = \frac{\tan 10^\circ \tan 20^\circ FC}{EF} = \frac{\tan 10^\circ \tan 20^\circ}{\tan 50^\circ}$.

2012…

2. B The interior angles of an equilateral triangle, square, regular pentagon are 60°, 90°, 108° respectively. The sum of the angles at a point is 360°. So $\theta = 360 - (60 + 90 + 108) = 102$.

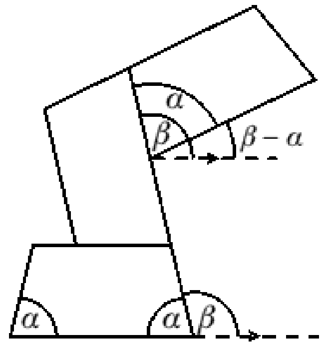
2013…

17. A Triangle PQR is equilateral so $\angle QPU = \angle UPT = \angle TPR = 20^\circ$. Triangle PUT is isosceles, so $\angle PUT = 80^\circ$. Let X be the midpoint of PQ and Y be the midpoint of UT.
Considering the right-angled triangle PXU gives $\cos 20^\circ = \dfrac{PX}{PU} = \dfrac{\frac{1}{2}}{PU}$, so $PU = \dfrac{1}{2\cos 20^\circ}$.
Considering the right-angled triangle PUY gives $\cos 80^\circ = \dfrac{UY}{PU}$, so $UY = PU \cos 80^\circ = \dfrac{\cos 80^\circ}{2 \cos 20^\circ}$. Therefore $UT = 2UY = \dfrac{2\cos 80^\circ}{2\cos 20^\circ} = \dfrac{\cos 80^\circ}{\cos 20^\circ}$.
{*Note that triangle UTS is a Morley triangle, named after the mathematician Frank Morley. His 1899 trisector theorem states that in any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle, in this case, triangle UTS.*}

24. E As they are vertically opposite, $\angle POQ = \angle SOR$. Let α denote the size of each of these. Applying the cosine rule to triangle SOR gives $8^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos \alpha$, therefore $40 \cos \alpha = -23$.
Similarly, from triangle POQ we obtain $x^2 = 4^2 + 10^2 - 2 \times 4 \times 10 \cos \alpha$. So $x^2 = 16 + 100 - 2 \times (-23) = 162$.
Hence $x = \sqrt{162} = \sqrt{81 \times 2} = 9\sqrt{2}$.

2014…

25. B Let the supplementary angle to α be β. Let tile 1 on the outside of the star polygon be horizontal. Counting anti-clockwise around the star polygon, tile 3 has an angle of elevation from the horizontal of $\beta - \alpha = 96\frac{2}{3}° - 83\frac{1}{3}° = 13\frac{1}{3}°$. As $360° \div 13\frac{1}{3}° = 27$, we need 27 pairs of tiles to complete one revolution. So there are 54 tiles in the complete pattern.



2015…

12. A Each of the three sides of triangle PQR is a tangent to the circle. Two tangents to a circle which meet at a point are of equal length. So QU and QS are of equal length. Similarly $RT = RS$. This means that $\angle QUS = \angle QSU = \frac{1}{2}(180 - \alpha)$ and also $\angle RTS = \angle RST = \frac{1}{2}(180 - \beta)$. At S we can consider the sum of the three angles, so $\frac{1}{2}(180 - \alpha) + \gamma + \frac{1}{2}(180 - \beta) = 180$. Simplifying gives $90 - \frac{1}{2}\alpha + \gamma + 90 - \frac{1}{2}\beta = 180$ and so $\gamma = \frac{1}{2}(\alpha + \beta)$.

2016…

4. C One angle in Alex's triangle is 80°. Let $\alpha°$ be the smaller of the other two angles so $(100 - \alpha)°$ is the third angle. The difference between these angles is then $(100 - 2\alpha)°$. Considering each option:
A: $100 - 2\alpha = 0$ gives both α and $100 - \alpha$ to be 50. This triangle is therefore isosceles and not scalene.
B: $100 - 2\alpha = 60$ gives α to be 20 and $100 - \alpha$ to be 80. This is again isosceles.
Option D gives angles of 80, 0 and 100. Option E gives angles of 80, −10 and 110. Neither of these cases forms a triangle.
C: $100 - 2\alpha = 80$ gives α to be 10 and $100 - \alpha$ to be 90. All three angles are different so this is the correct option.

15. B A non-regular hexagon can be drawn on the diagram as shown. Three of the exterior angles of the hexagon are then 55°, 60° and 65°. Since corresponding angles on parallel lines are equal, the other three exterior angles are $p°$, $q°$ and $r°$. The total of the exterior angles of any polygon is 360°. Hence $p + q + r + 55 + 60 + 65 = 360$ and so $p + q + r = 180$.