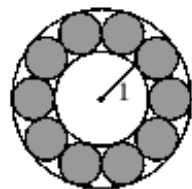




21. The diagram shows ten equal discs that lie between two concentric circles – an inner circle and an outer circle. Each disc touches two neighbouring discs and both circles. The inner circle has radius 1.



What is the radius of the *outer* circle?

- A  $2 \tan 36^\circ$       B  $\frac{\sin 36^\circ}{1 - \sin 36^\circ}$       C  $\frac{1 + \sin 18^\circ}{1 - \sin 18^\circ}$       D  $\frac{2}{\cos 18^\circ}$       E  $\frac{9}{5}$

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21. C As there are 10 discs, the adjacent lines drawn from the centre of the inner circle to the centre of each disc are separated by an angle of  $36^\circ$ . The line  $OB$  is a tangent to both the disc with centre  $A$  and the disc with centre  $C$ . So the points  $A$ ,  $B$  and  $C$  lie on a straight line as angles  $OBA$  and  $OBC$  are both  $90^\circ$ .

In the second diagram, from triangle  $OAB$  we

have  $\sin 18^\circ = \frac{r}{1+r}$  which rearranges to

$$\frac{\sin 18^\circ}{1 - \sin 18^\circ} = r.$$

The radius of the outer circle is

$$1 + 2r = 1 + \frac{2 \sin 18^\circ}{1 - \sin 18^\circ} = \frac{1 + \sin 18^\circ}{1 - \sin 18^\circ}.$$

