



20. A square ink pad has sides of length 1 cm. It is covered in black ink and carefully placed in the middle of a piece of white paper. The square pad is then rotated  $180^\circ$  about one of its corners so that all of the pad remains in contact with the paper throughout the turn. The pad is then removed from the paper. What area of paper, in  $\text{cm}^2$ , is coloured black?

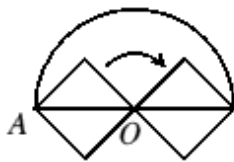
A  $\pi + 2$       B  $2\pi - 1$       C 4      D  $2\pi - 2$       E  $\pi + 1$

1590



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20. E



Let the corner of the square about which it is rotated be  $O$  and the opposite vertex of the square be  $A$ . As the square is rotated through  $180^\circ$  about  $O$ , the vertex  $A$  travels along a semicircle whose centre is  $O$ . The area coloured black by the ink is then formed from two half squares and a semicircle. The square has side-length 1, so  $OA = \sqrt{2}$ . The total area of the two half squares and the semicircle is  $2 \times (\frac{1}{2} \times 1 \times 1) + \frac{1}{2} \times \pi \times (\sqrt{2})^2$  which is  $1 + \pi$ .