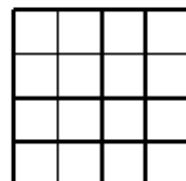




19. The 16 small squares shown in the diagram each have a side length of 1 unit. How many pairs of vertices are there in the diagram whose distance apart is an integer number of units?



- A 40 B 64 C 108 D 132 E 16

1389



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19. C There are 25 vertices in the diagram. Each vertex is part of a row of 5 vertices and a column of 5 vertices. Each vertex is therefore an integer number of units away from the 4 other vertices in its row and from the other 4 vertices in its column. This appears to give $25 \times (4 + 4) = 200$ pairs. However, counting in this manner includes each pair twice so there are only 100 different pairs.
- By using the Pythagorean triple 3, 4, 5, each corner vertex is five units away from two other non-corner vertices, giving another 8 pairs. No other Pythagorean triples include small enough numbers to yield pairs of vertices on this grid.
- Thus the total number of pairs is 108.